

Small-Scale Λ CDM Problems

Federico Lelli (Arcetri Astrophysical Observatory)

Download Slides at www.lectifederico.com/students

Largely based on the following review papers:

Bullock & Boylan-Kolchin (2017), ARA&A

McGaugh, Lelli, Li, Schombert (2020), IAU Symposium 353

Lelli (2022), Nature Astronomy

Lelli (2023), IAU Symposium 379

Testing Λ CDM at different astronomical scales

Galaxy Scales (~1-100 kpc)

Andromeda (spiral galaxy)



Messier 87 (elliptical galaxy)

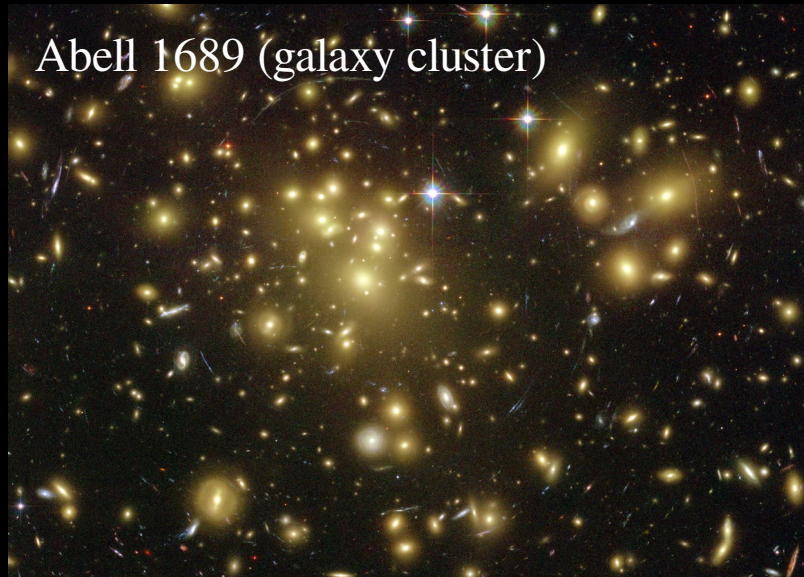


Group/Cluster Scales (~1-5 Mpc)

Stephan's Quintet (galaxy group)

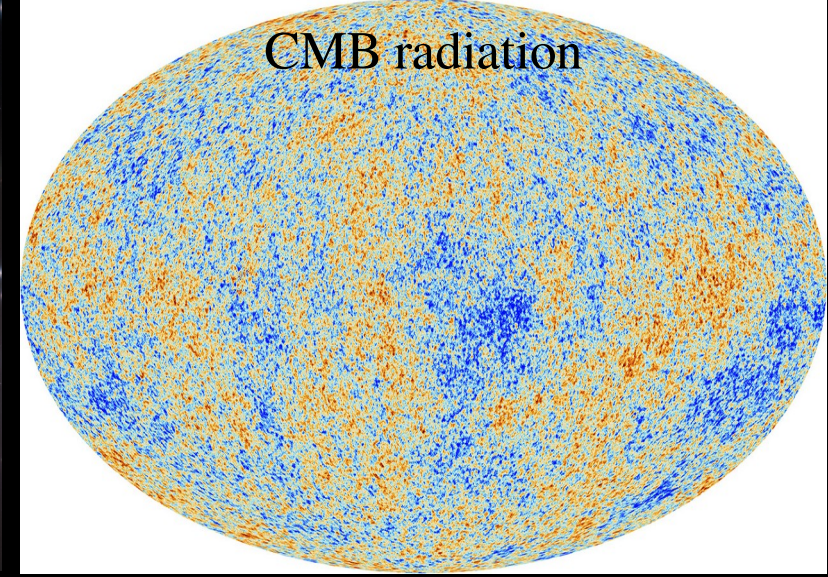


Abell 1689 (galaxy cluster)

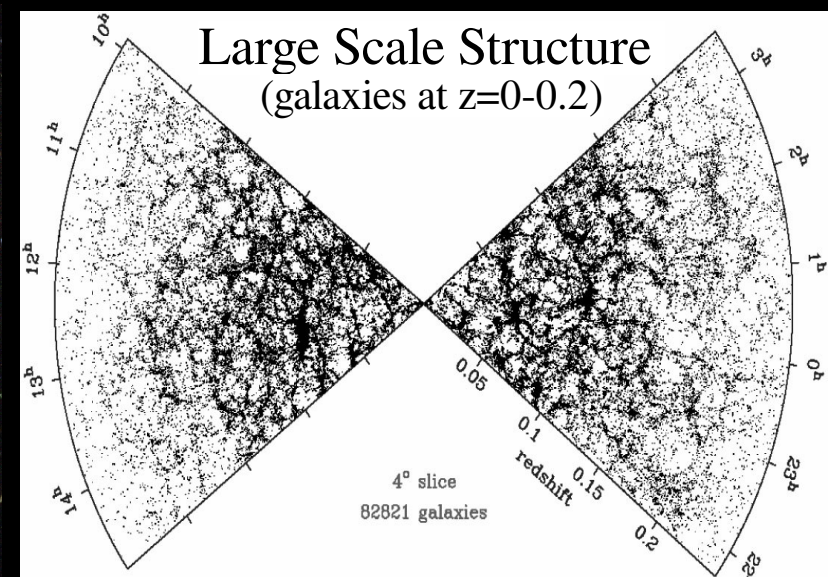


Cosmological Scales (>100 Mpc)

CMB radiation



Large Scale Structure
(galaxies at $z=0-0.2$)



Small-Scale Λ CDM Problems

1. Missing Satellites
2. Cusp vs Core
3. Too-Big-To-Fail
4. Planes of Satellites
5. Dynamical Regularities

Small-Scale Λ CDM Problems

1. Missing Satellites

2. Cusp vs Core

3. Too-Big-To-Fail

“Classical” problems
(~25 years old)

4. Planes of Satellites

5. Dynamical Regularities

“New” problems
(~10 years old)

Why are these problems interesting?

1. Baryonic physics inside CDM halos?

Gas cooling, star formation, stellar feedback, etc.

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2. New physics in the dark sector?

Self-interacting DM, fuzzy DM, superfluid DM, etc.

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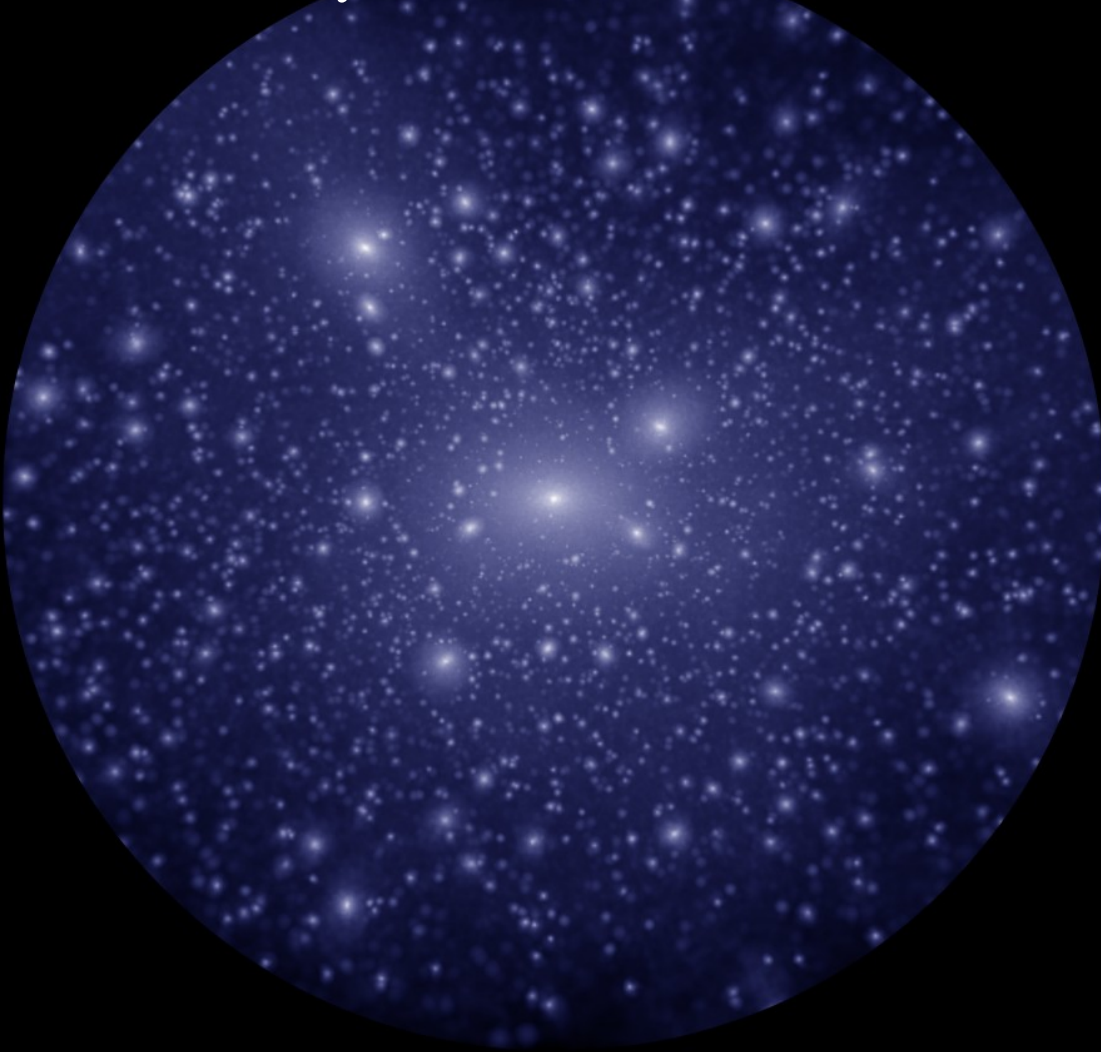
3. New dynamical laws rather than DM?

Milgromian dynamics (MOND), modified gravity, etc.

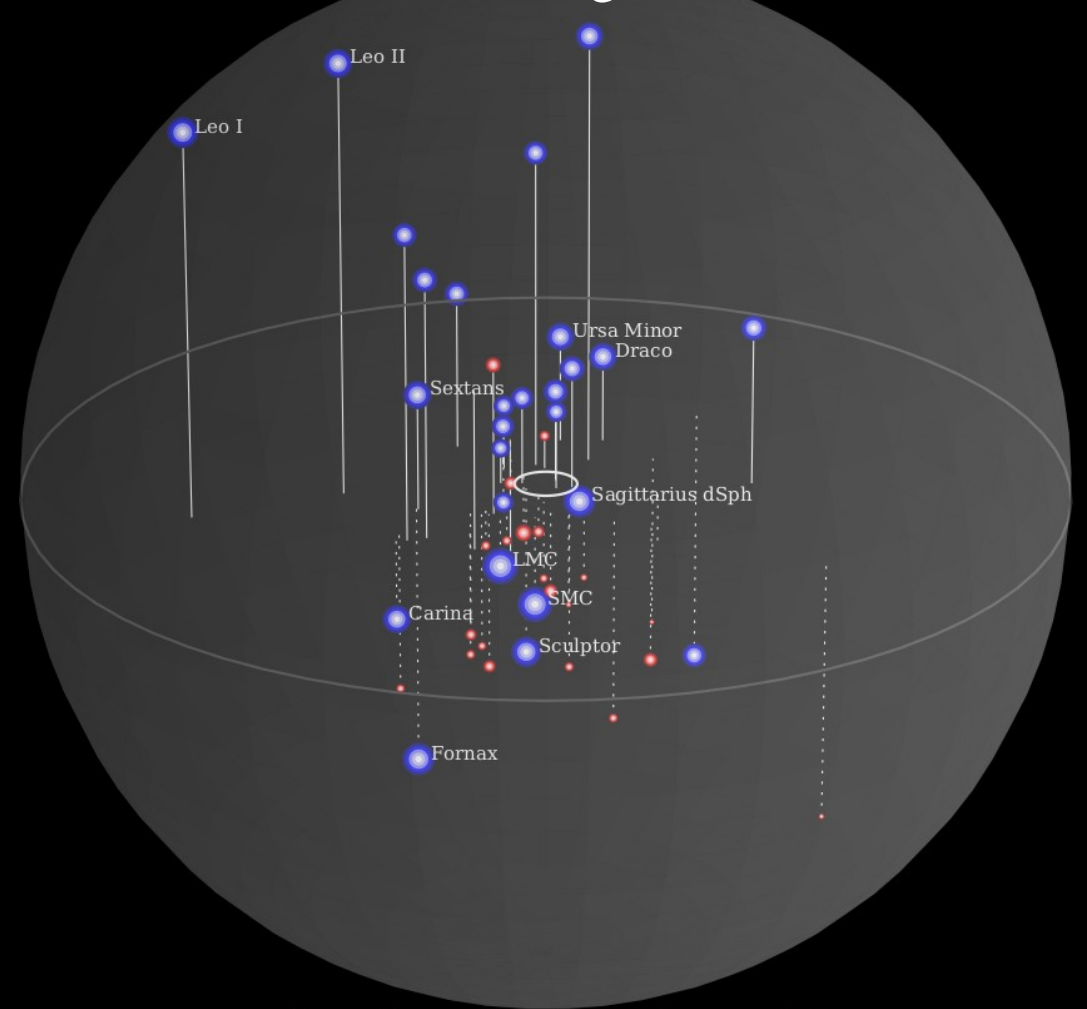
1. Missing Satellites Problem (Mass Function Problem)

The Missing Satellites Problem

Λ CDM N-body Simulation: ~1000s sub-halos



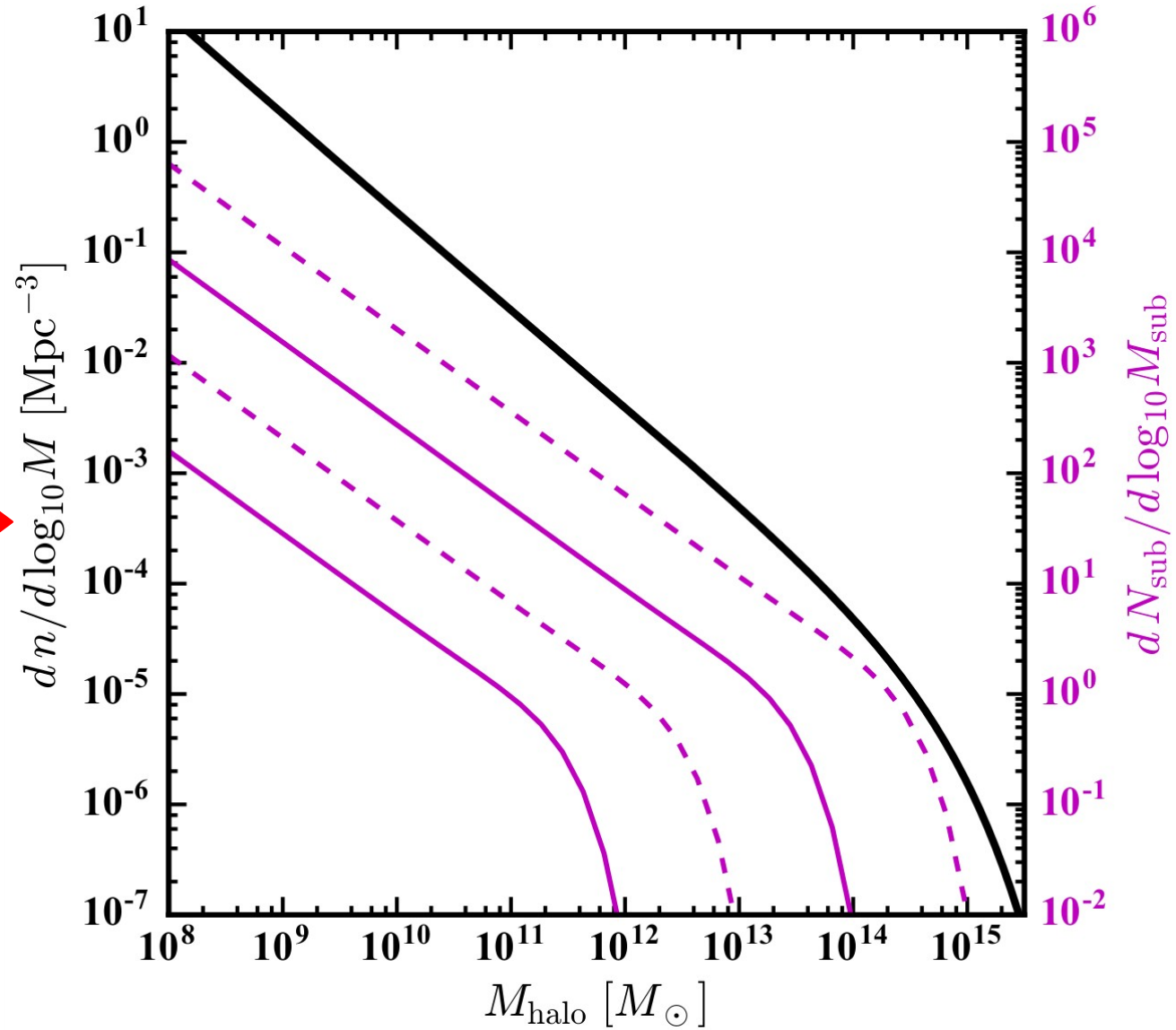
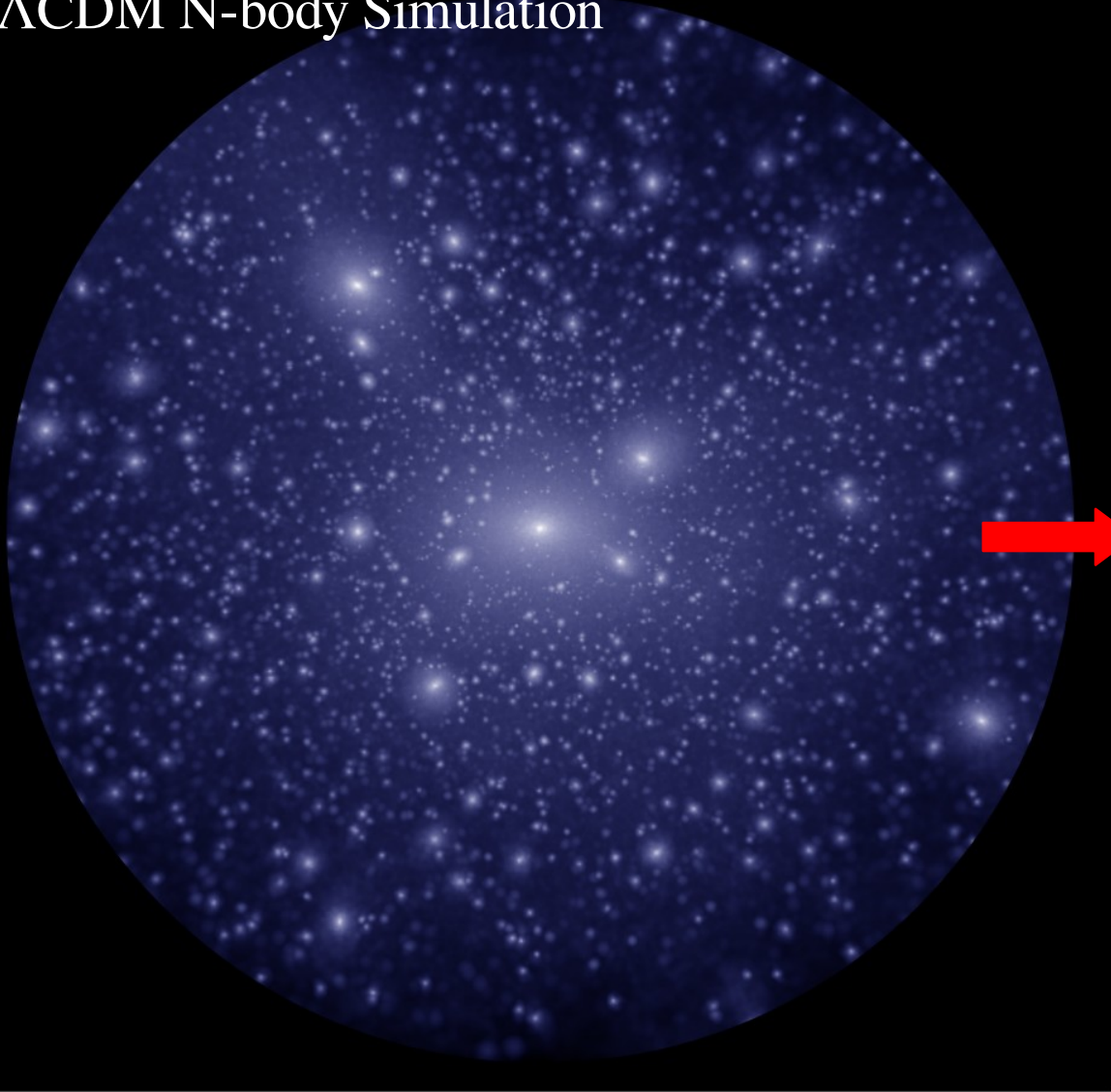
Observations: ~50 satellite galaxies around MW



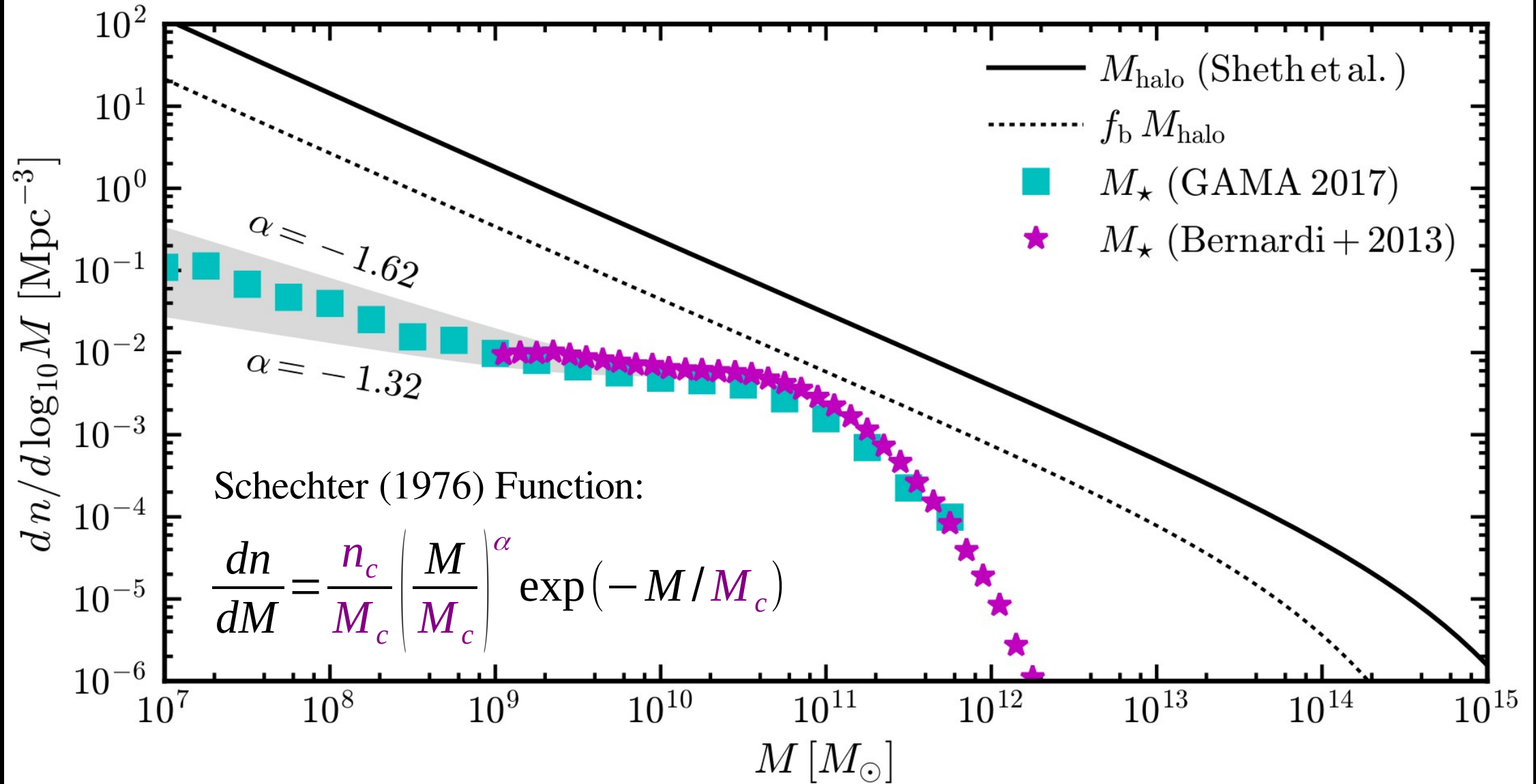
Pawlowski/Bullock/Boylan-Kolchin

The Halo Mass Function

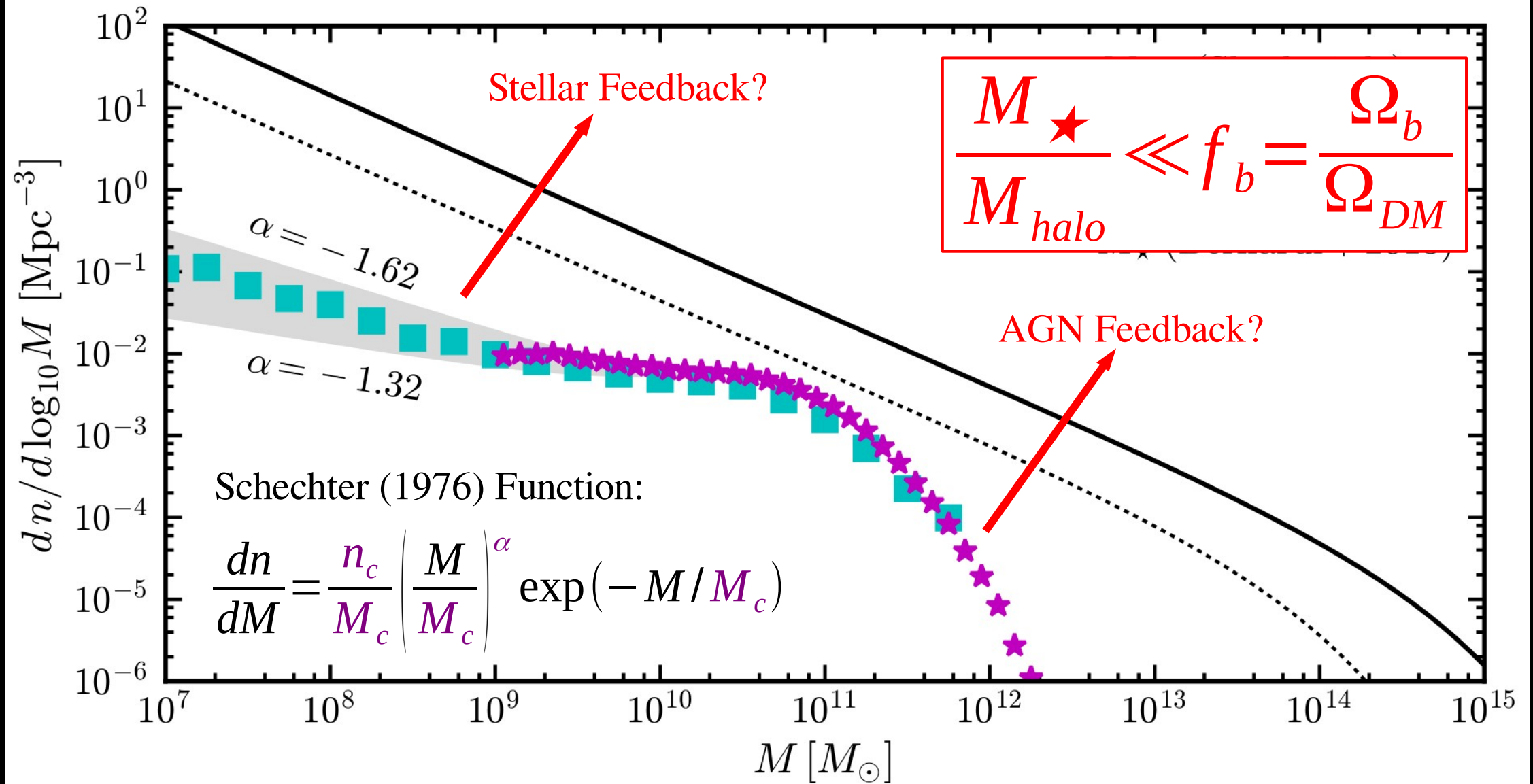
Λ CDM N-body Simulation



Stellar versus Halo Mass Function

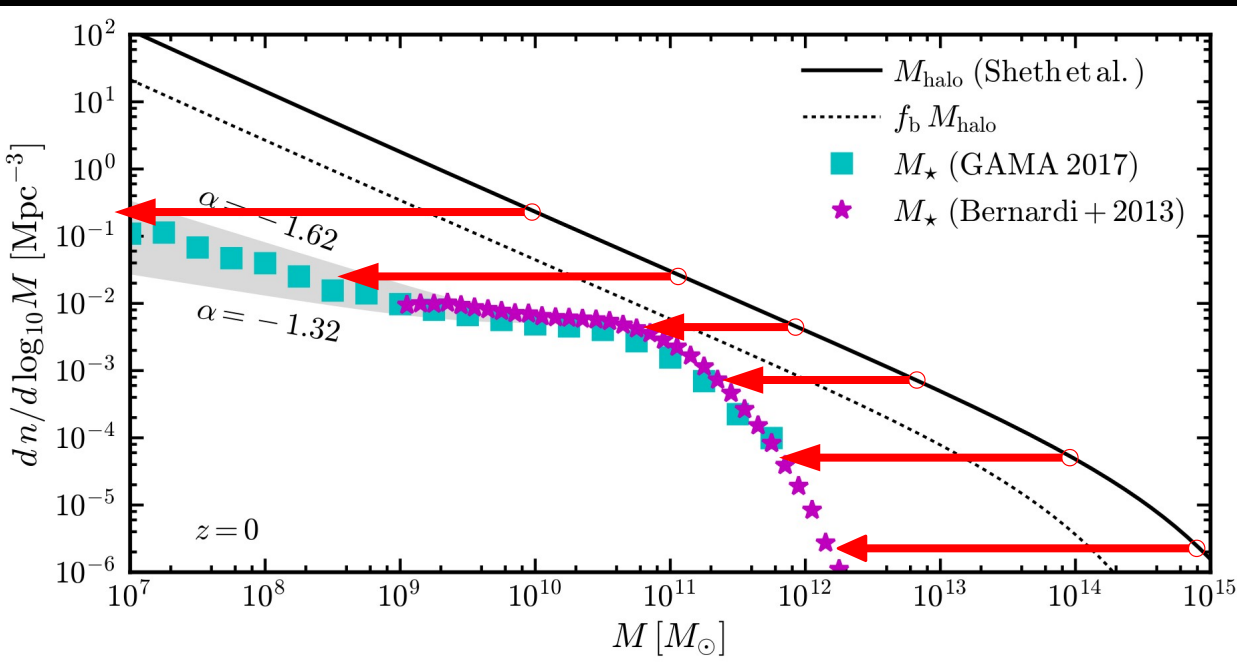


Stellar versus Halo Mass Function



The Stellar Mass – Halo Mass Relation

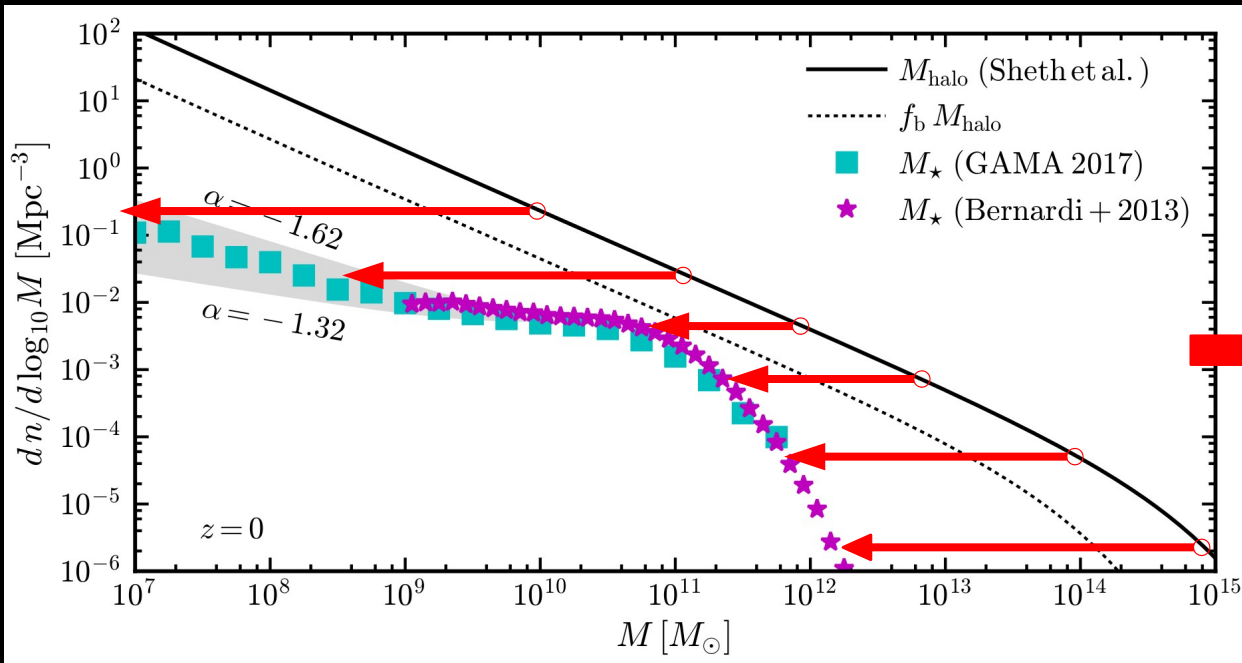
Abundance Matching: “brute-force” method that require no knowledge of baryonic physics



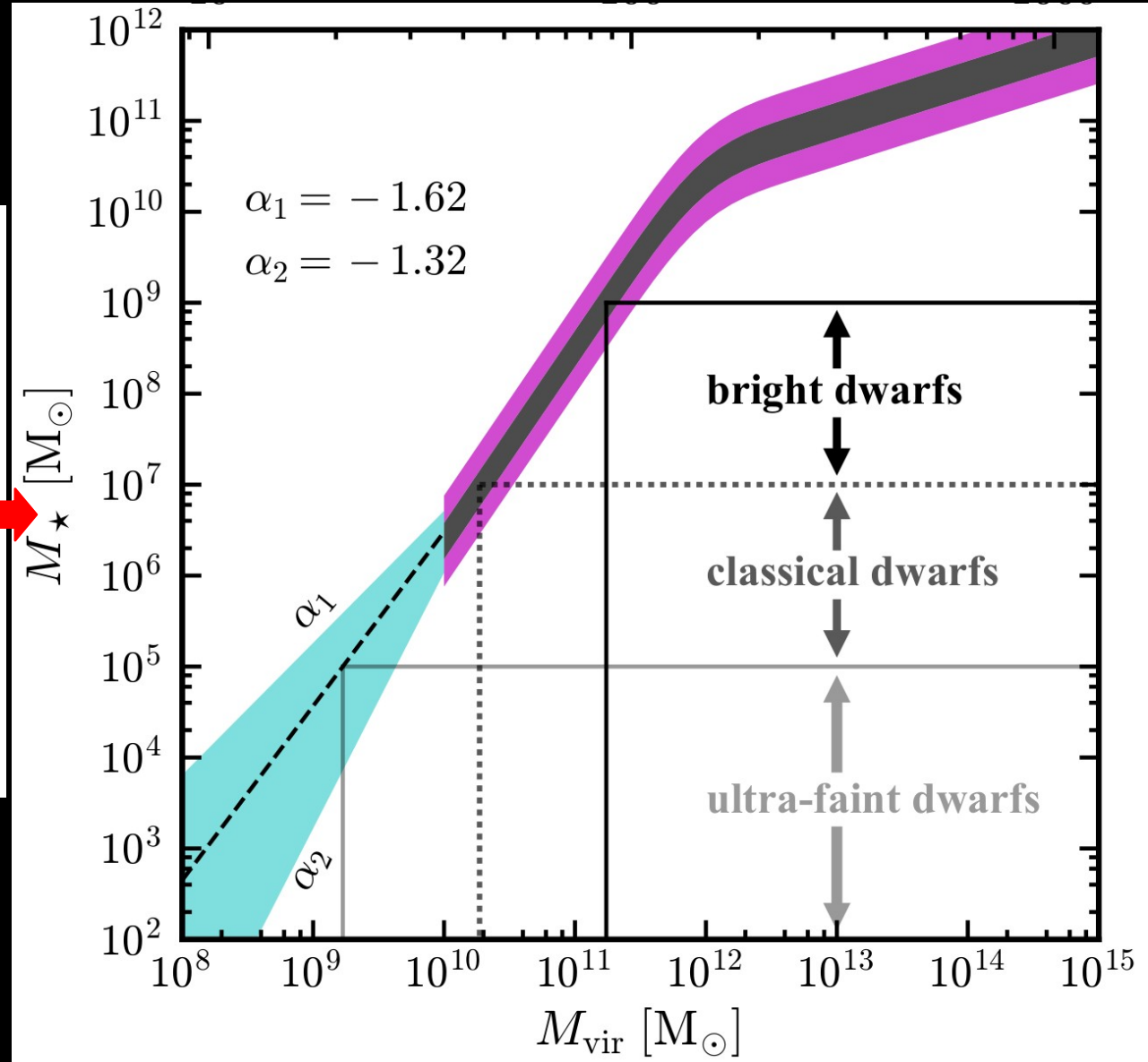
Given a volume of the Universe (say 1 Gpc^3), assign the most massive halo (in a simulation) to the most massive galaxy (in a survey).

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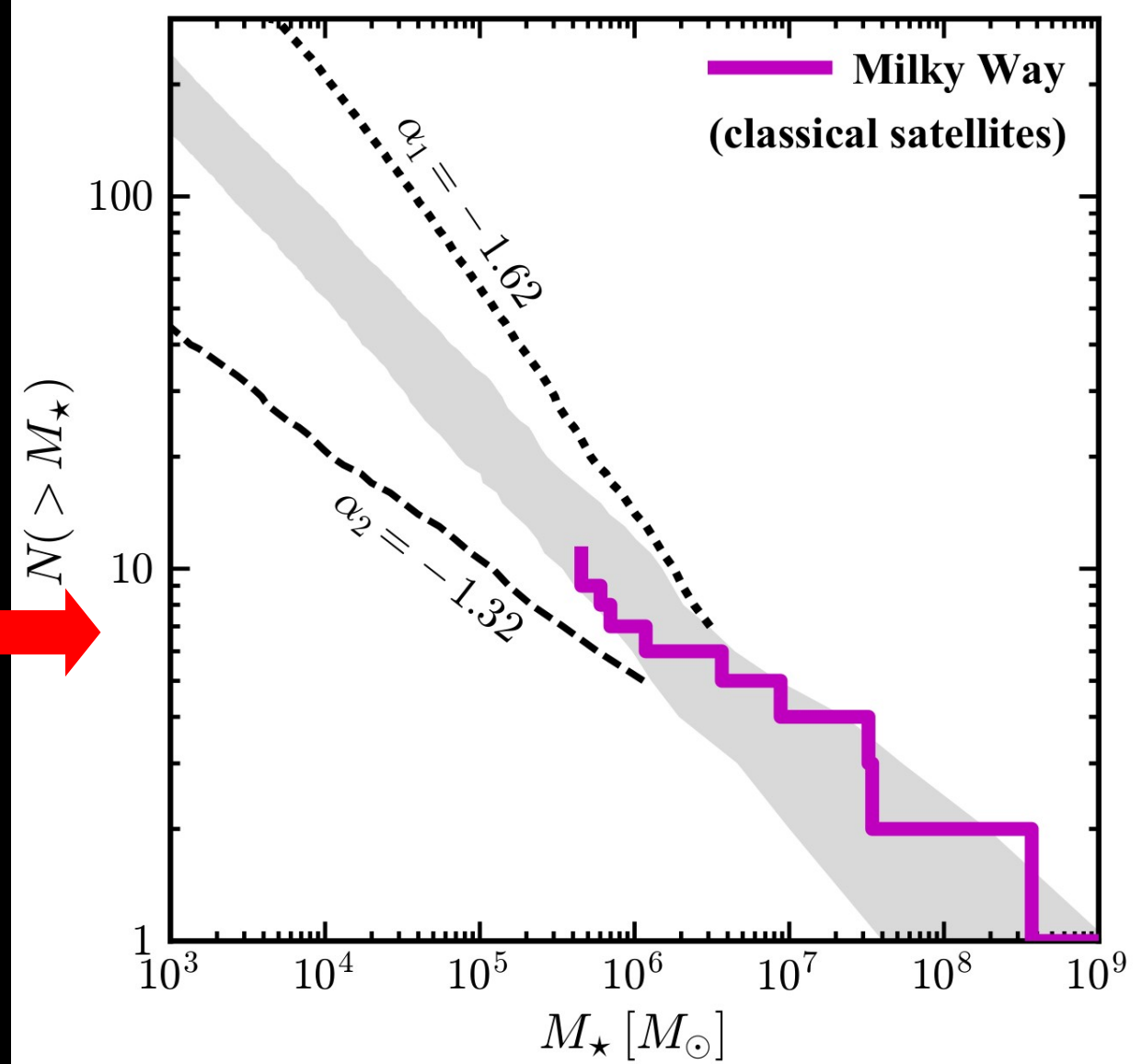
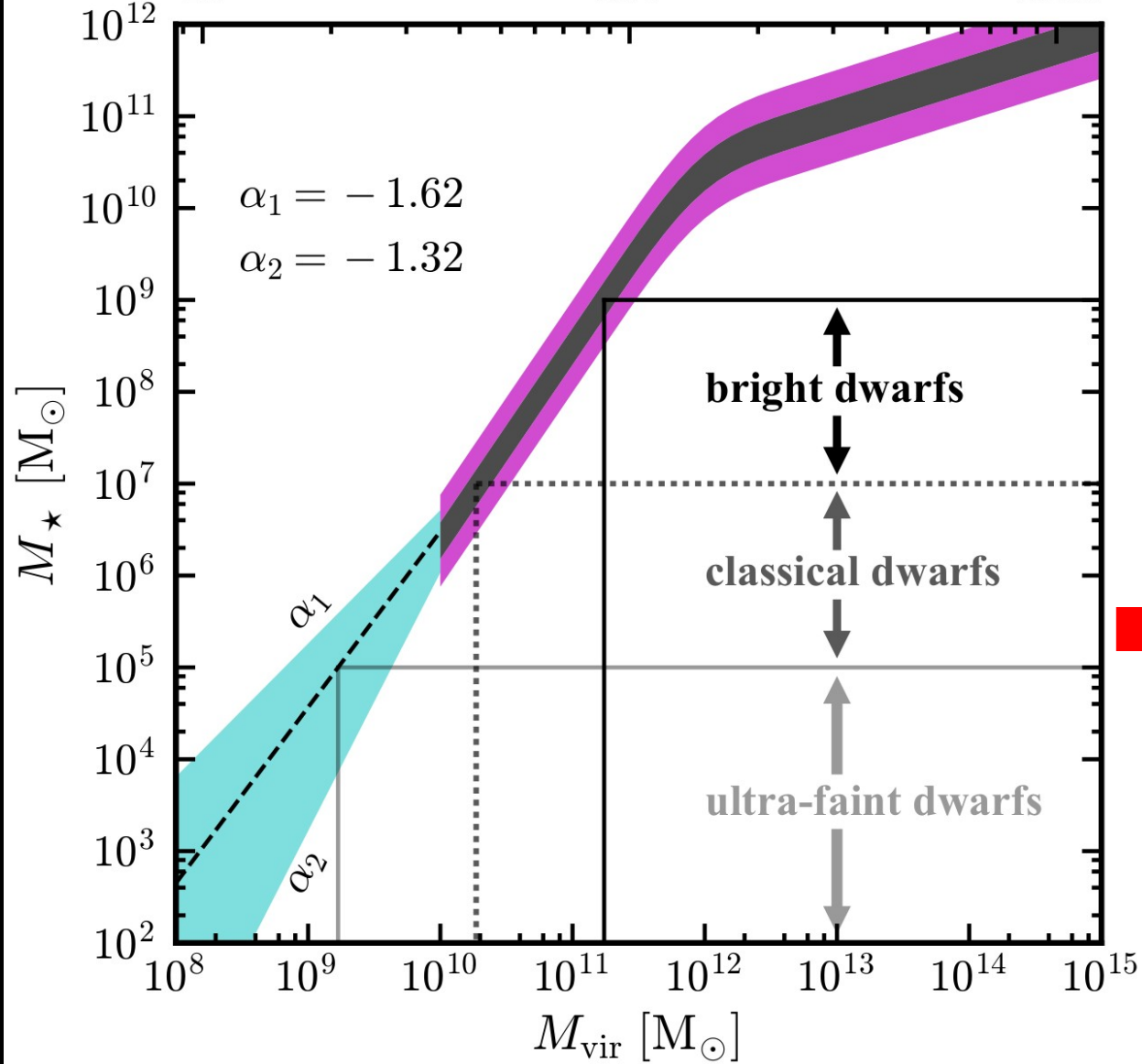
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Abundance Matching \leftrightarrow Missing Satellites



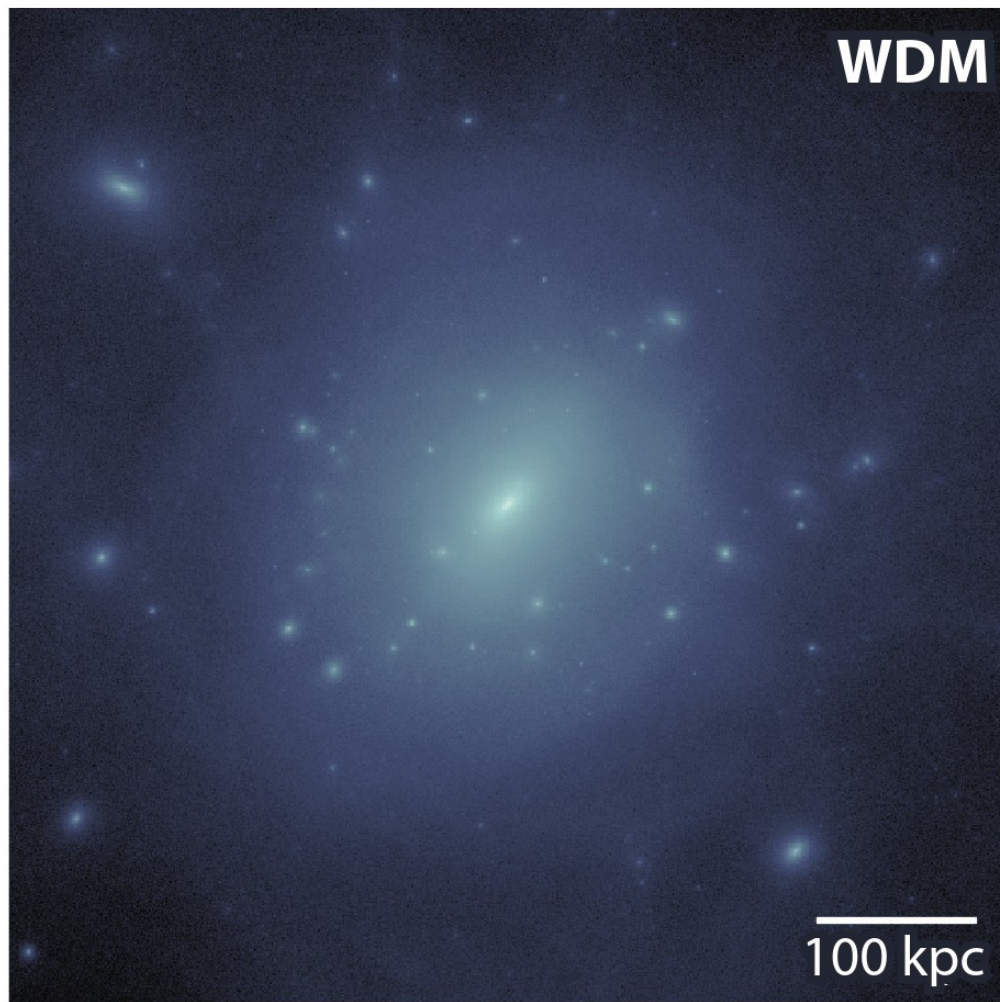
Possible Solutions to the Missing Satellite (Mass Function) Problem

1. **Baryonic physics:** stellar & AGN feedback somehow reduce the amount of collapsed baryons (stars & cold gas) in galaxies
→ most baryons are in a hot “invisible” form (missing baryons)

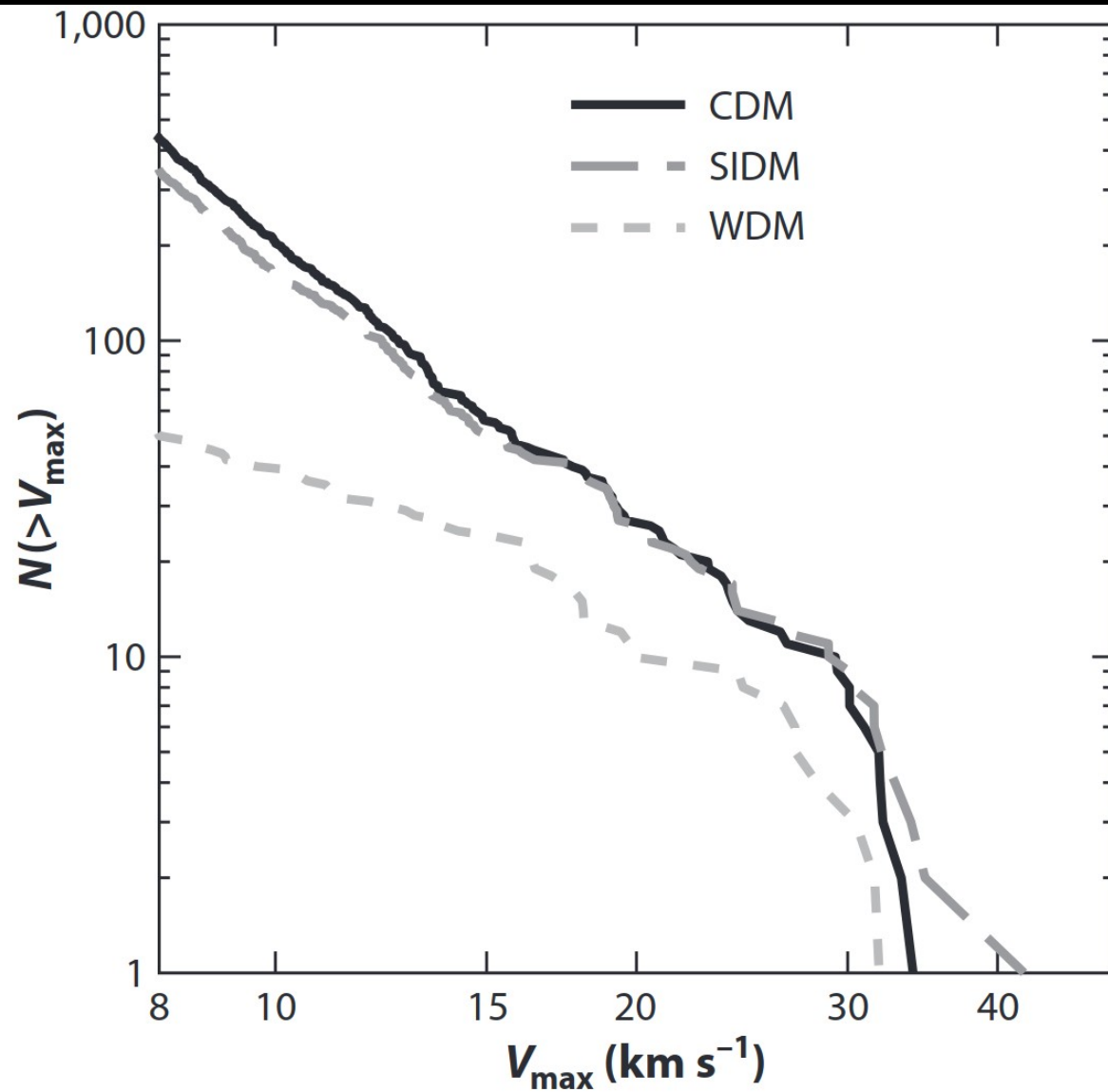
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→ reduce number of predicted low-mass DM halos

Smaller number of sub-halos in WDM



From Bullock & Boylan-Kolchin (2017)



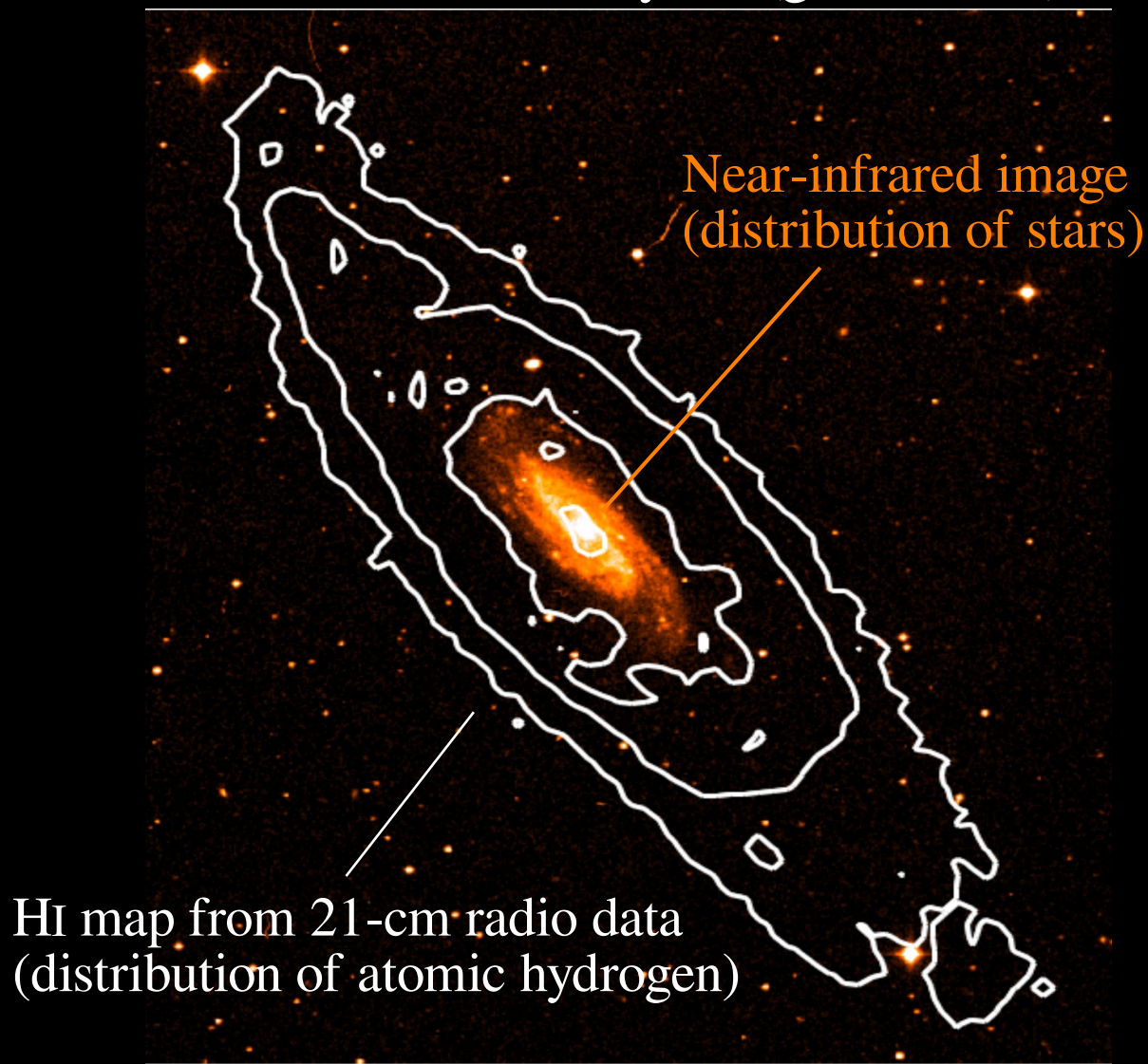
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2. Cusp vs Core Problem (Rotation Curves Problem)

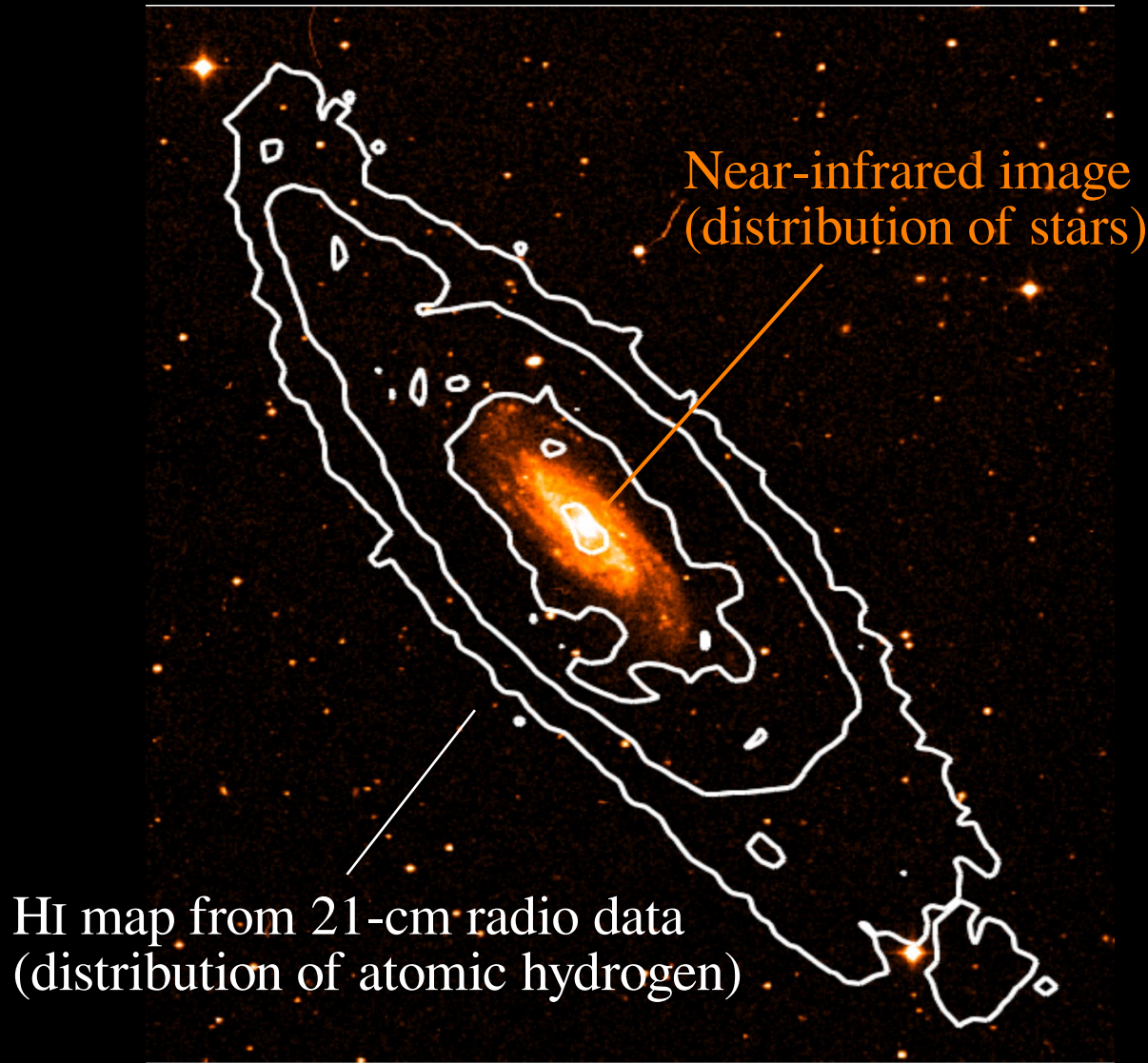
Rotation Curves of Disk Galaxies

Distribution of baryons (gas & stars)

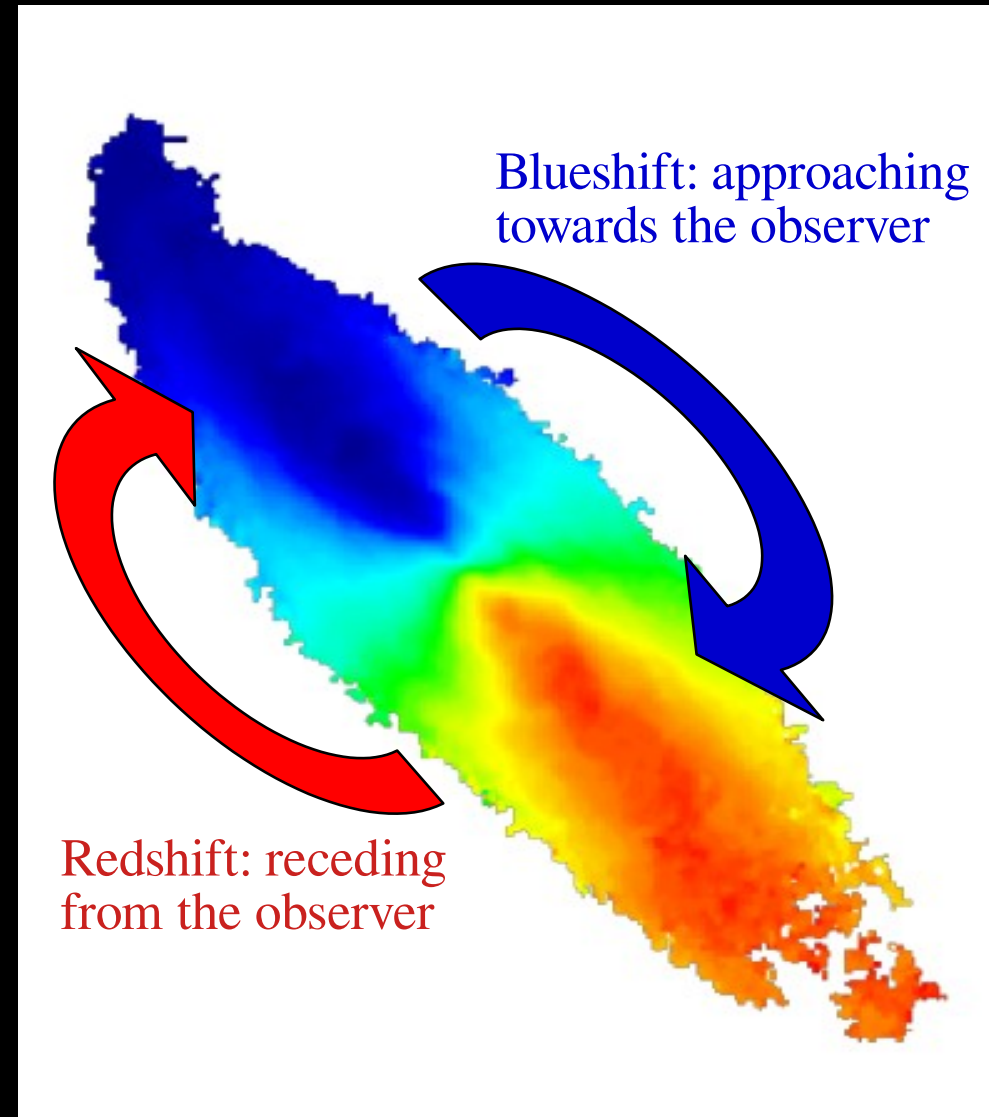


Rotation Curves of Disk Galaxies

Distribution of baryons (gas & stars)



Velocity along the Line of Sight (LoS)

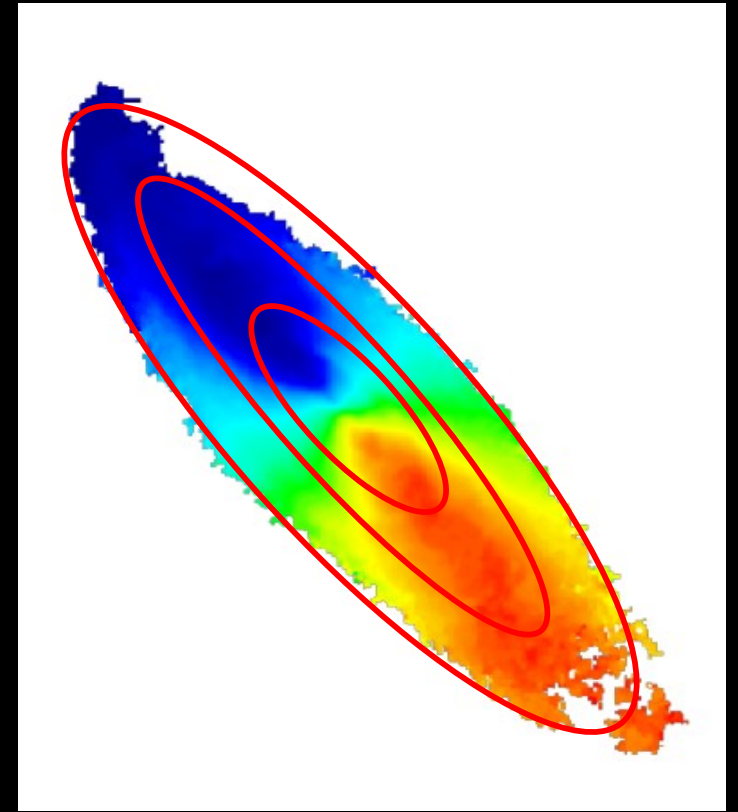
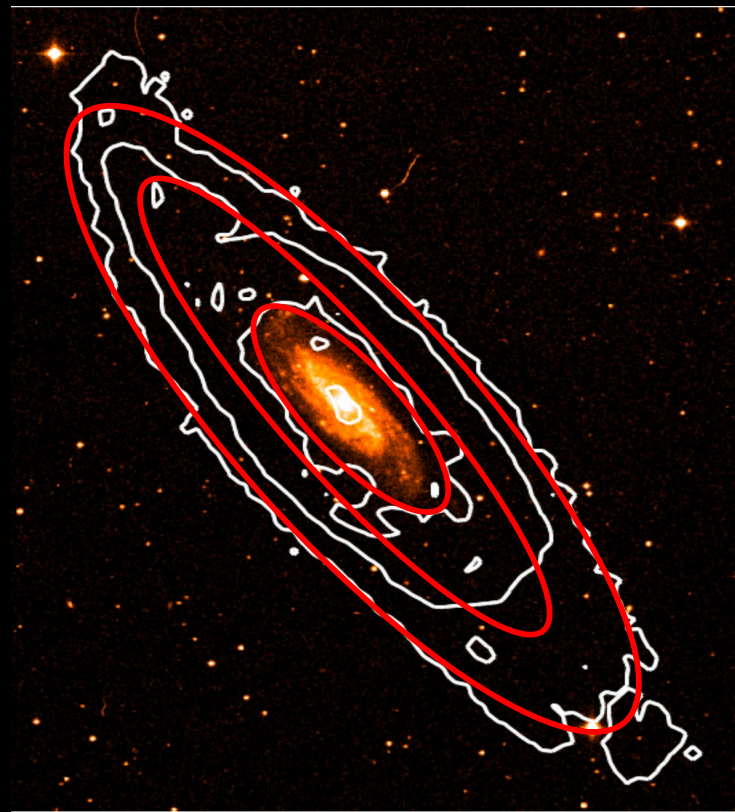
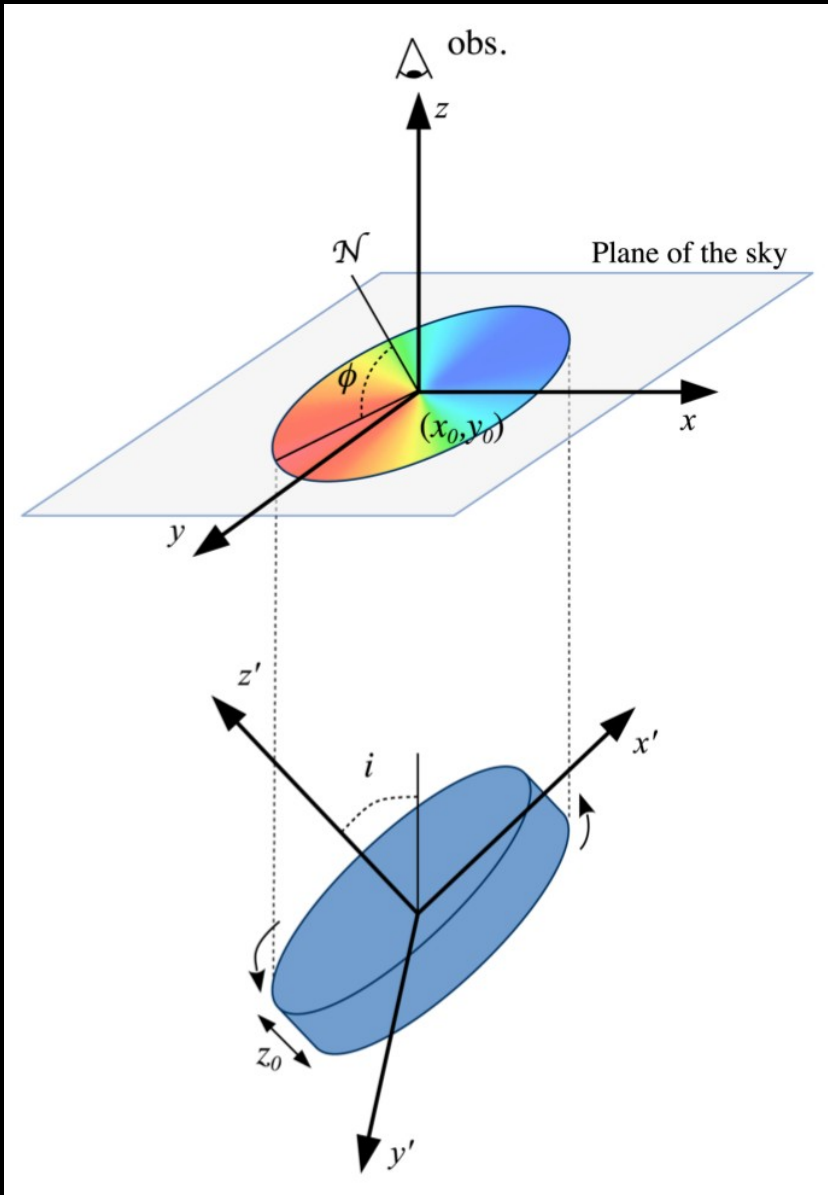


Deprojection from sky-plane to galaxy-plane

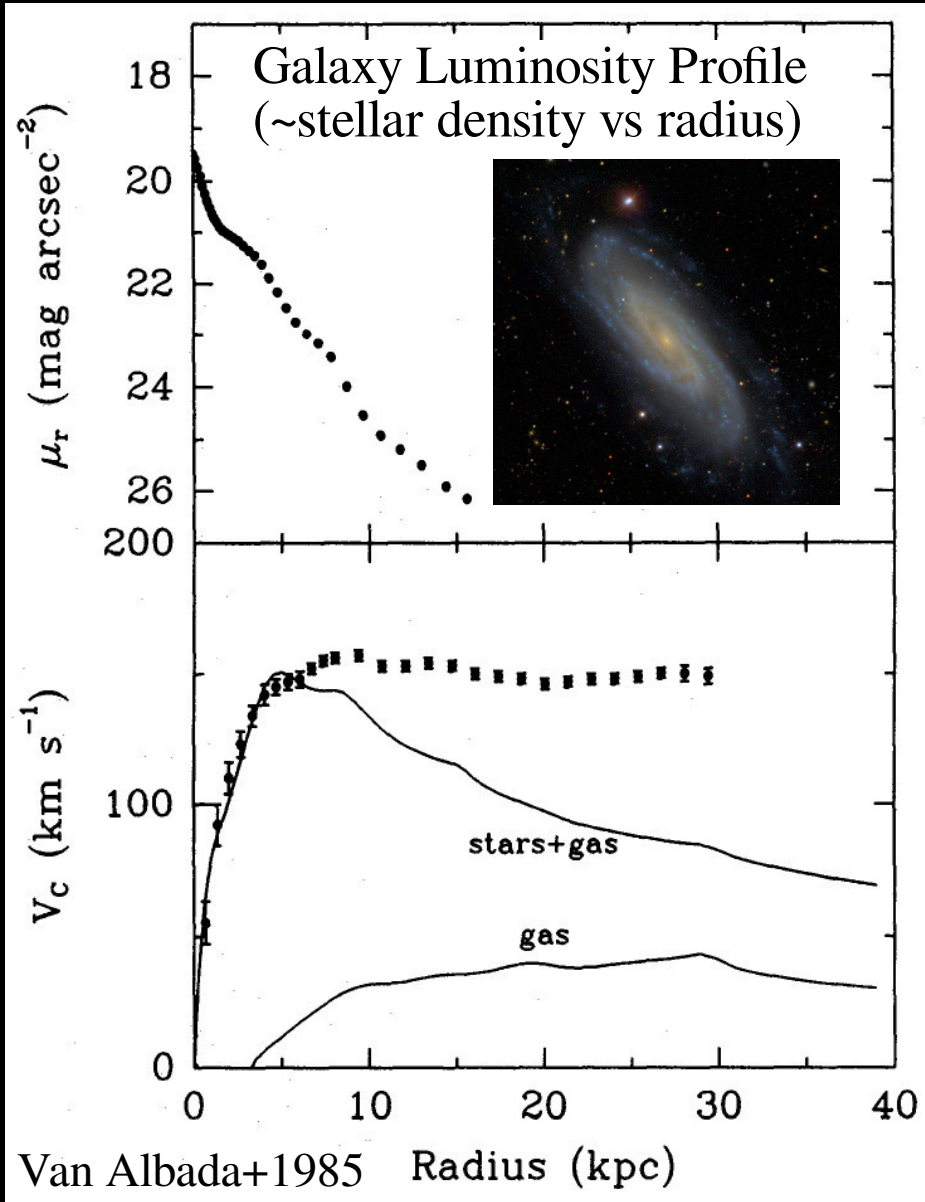
For a thin disk ($V_{\text{rot}} \gg \sigma_v$) projected on the sky:

$$V_{\text{LoS}}(x, y) = V_{\text{sys}} + V_{\text{rot}}(R) \sin(i) \cos(\theta)$$

$\cos(\theta) = f(x_0, y_0, PA) \rightarrow$ Disk Geometric Parameters



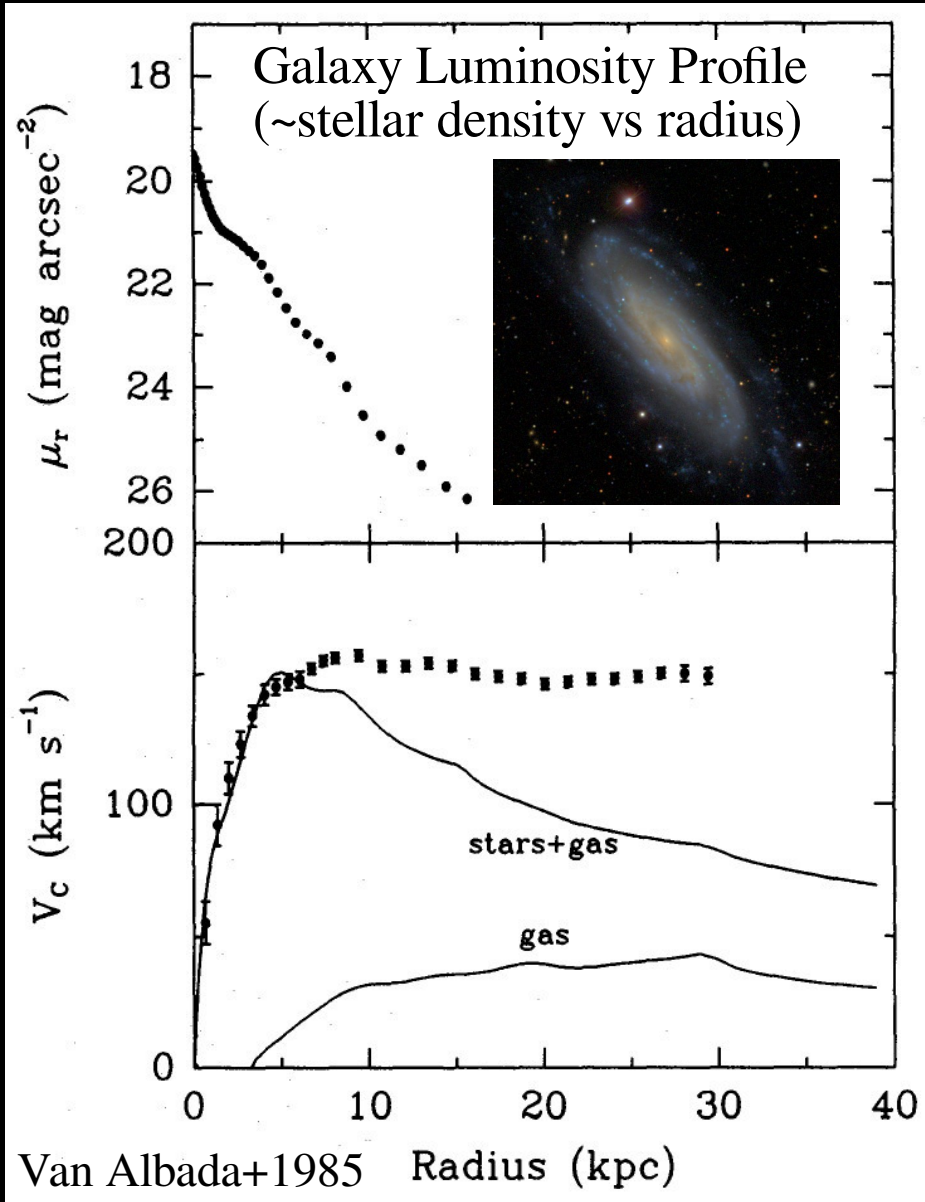
Mass Models of Disk Galaxies



- Solve (numerically) Poisson's equation in cylindrical coordinates for each component ($i = \text{stars or gas}$):

$$\nabla^2 \Phi_i(R, z) = 4\pi G \rho_i(R, z)$$

Mass Models of Disk Galaxies



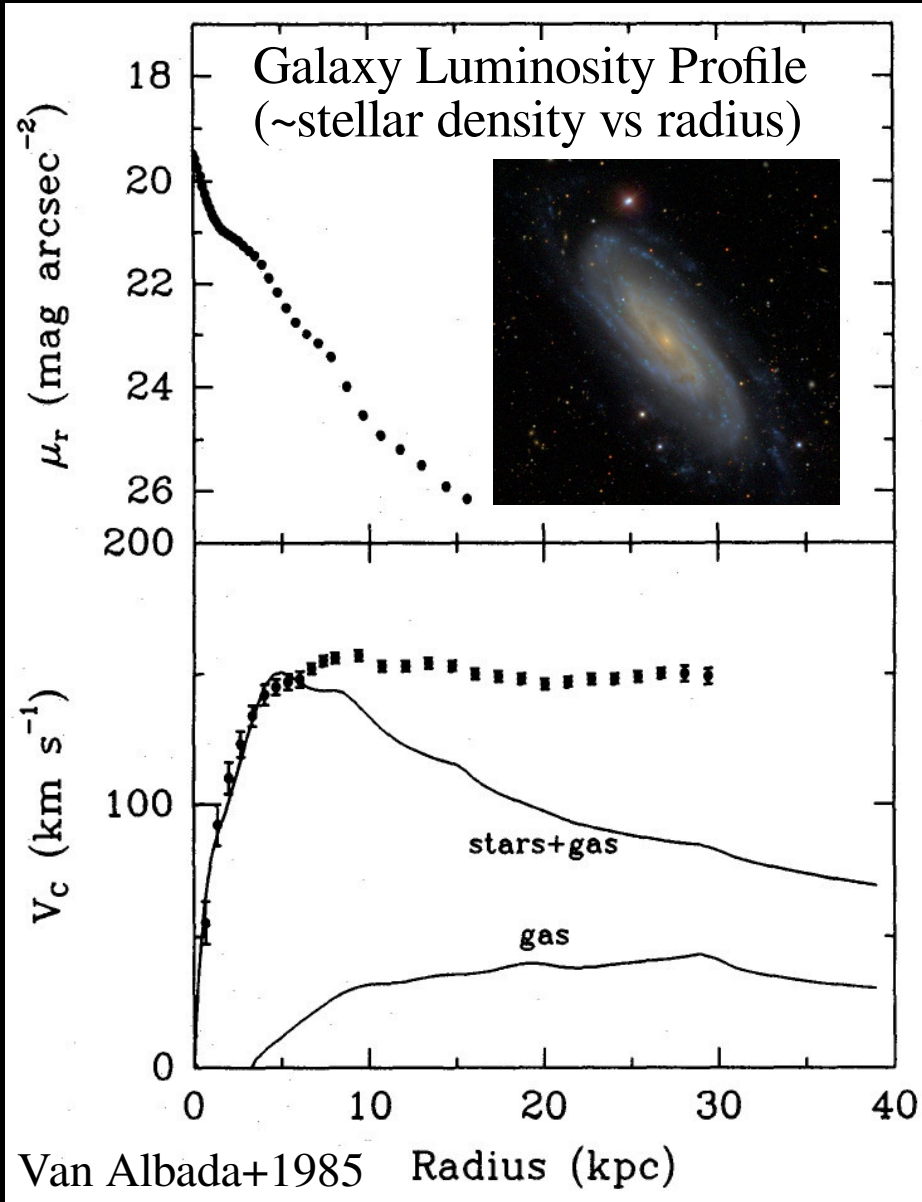
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- Find expected circular velocity in disk mid-plane:

$$\frac{V_i^2(R, z=0)}{R} = - \frac{\partial \Phi_i(R, z=0)}{\partial R}$$

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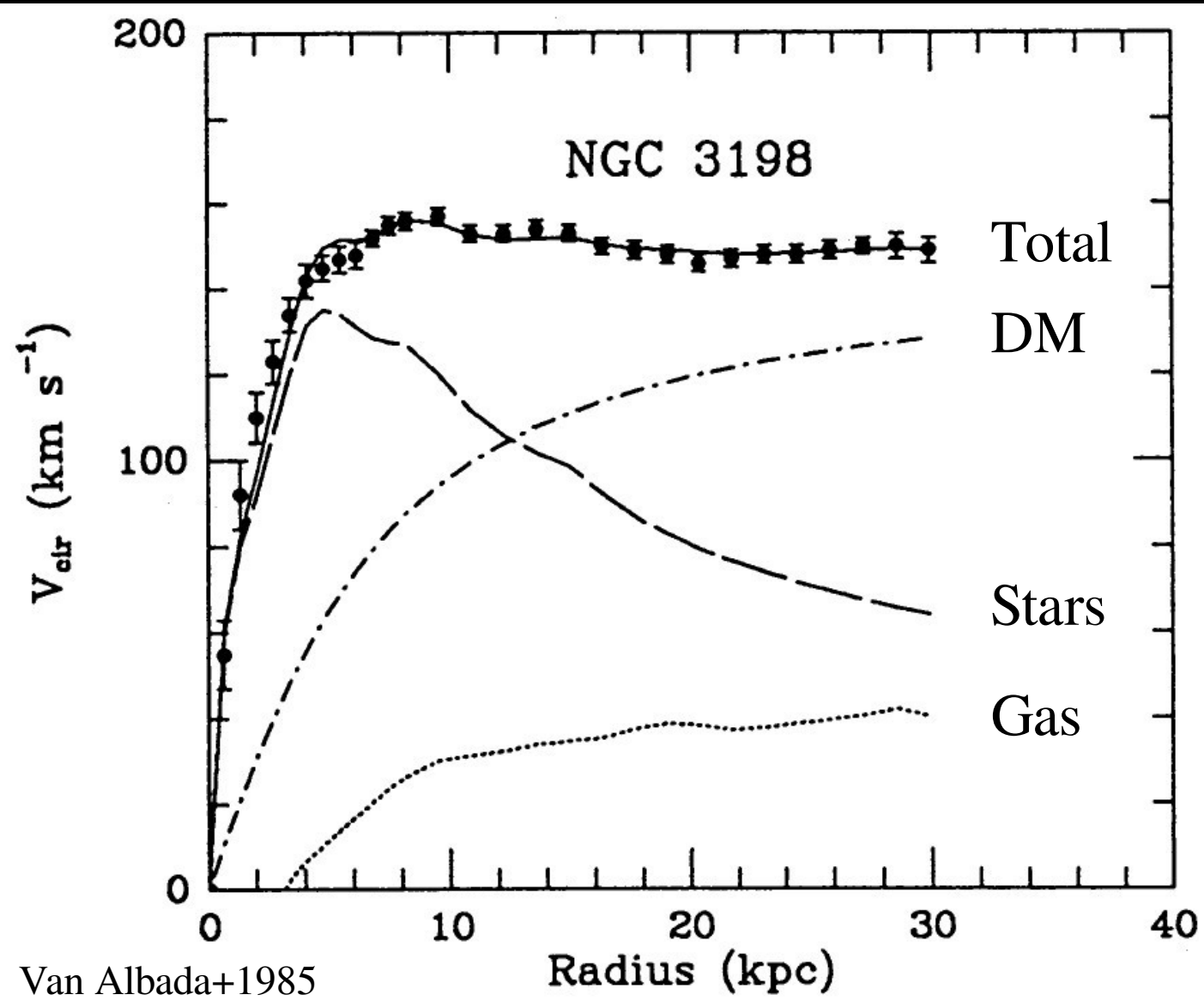
- Sum over gravitational fields ($g_i = V_i^2/R$):

$$V_b^2(R) = Y_{\star} V_{\star}^2(R) + Y_g V_g^2(R)$$

$Y_s = M_s/L$ estimated from stellar population models

$Y_g =$ known for HI from atomic physics (spin-flip) + small corrections for H $_2$, He, heavier elements

Mass Model with a Dark Matter Halo



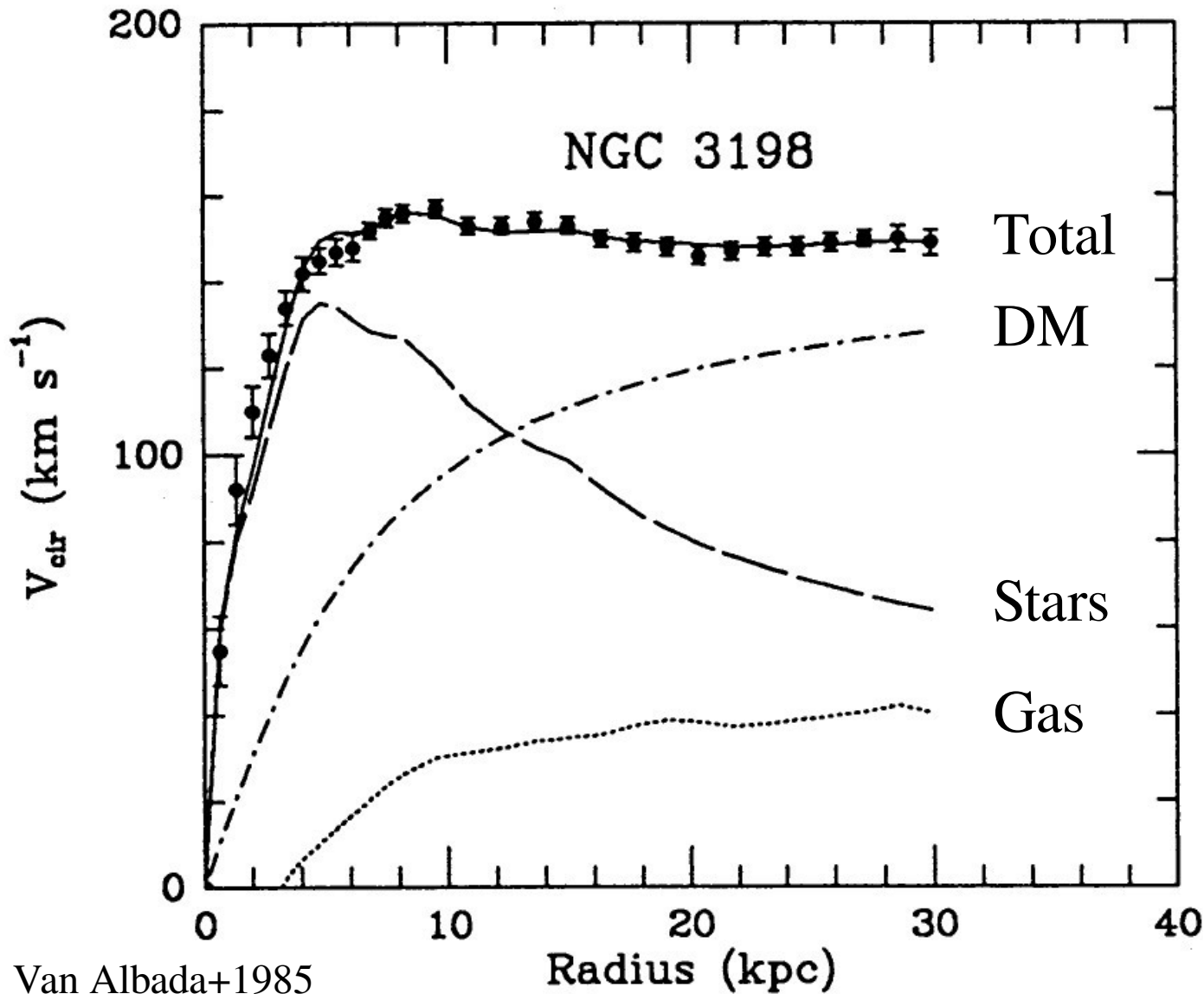
- Assume spherical DM halo profile:

$$\rho_{DM} = \rho(r; \rho_c, r_c)$$

- Add it together with the baryons:

$$V_c^2 = Y_{\star} V_{\star}^2 + Y_g V_g^2 + V_{DM}^2(\rho_c, r_c)$$

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For spiral galaxies like the Milky Way, baryons dominate in the inner parts while DM is needed in the outer regions

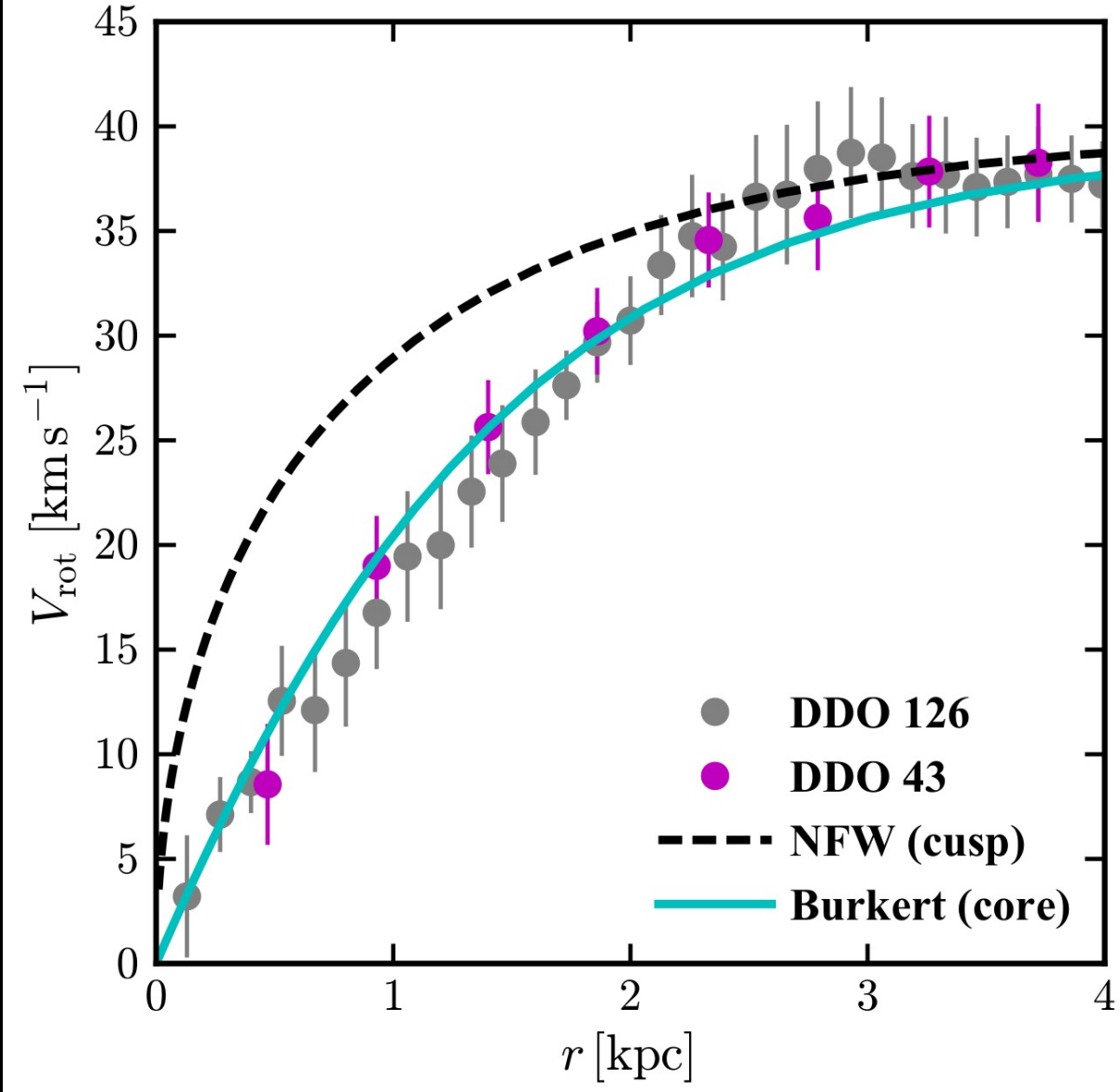
$$V_c^2(R) = V_b^2(R) + V_{DM}^2(R) = \text{const}$$

Why are rotation curves flat? Unclear!

Fine-tuning issue: “Disk-halo conspiracy”

(van Albada & Sancisi 1986)

Cusp-Core Problem in Dwarf Galaxies



NFW profile (from N-body sims):

$$\rho_{DM}(r) = \frac{4\rho_c}{(r/r_c)(1+r/r_c)^2}$$

Burkert profile (empirical):

$$\rho_{DM}(r) = \frac{\rho_c}{(1+r/r_c)[1+(r/r_c)^2]}$$

Possible Solutions to the Cusp-Core (Rotation Curve) Problem

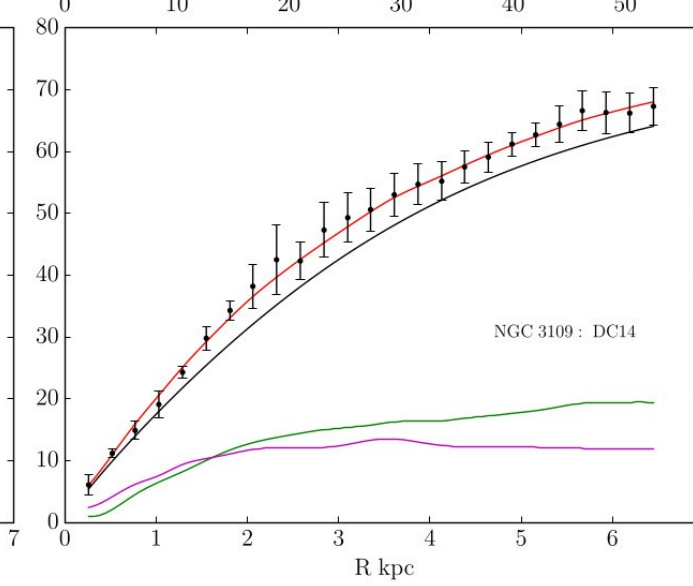
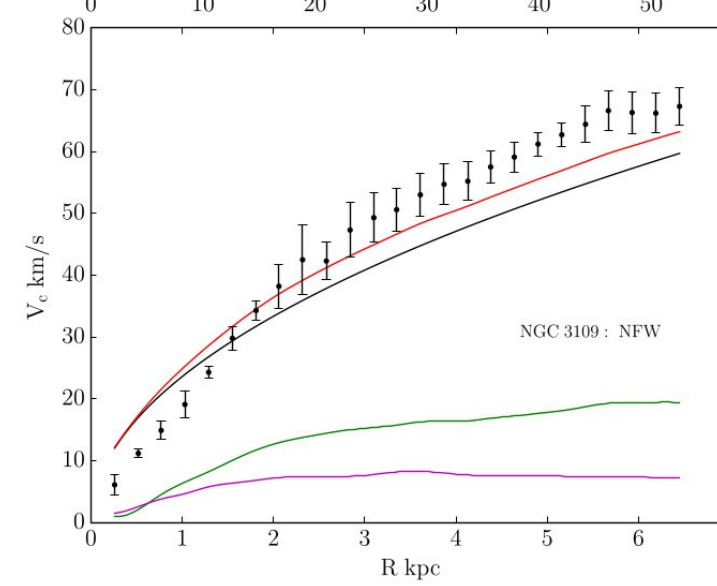
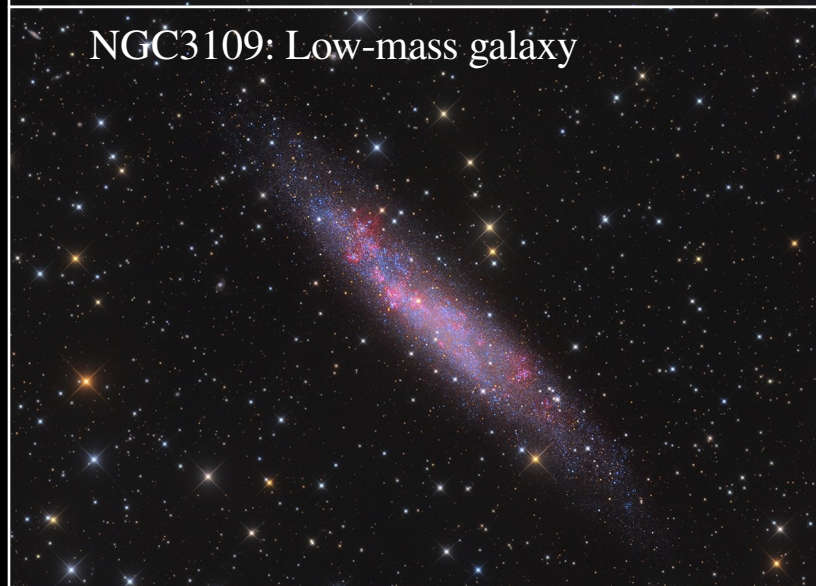
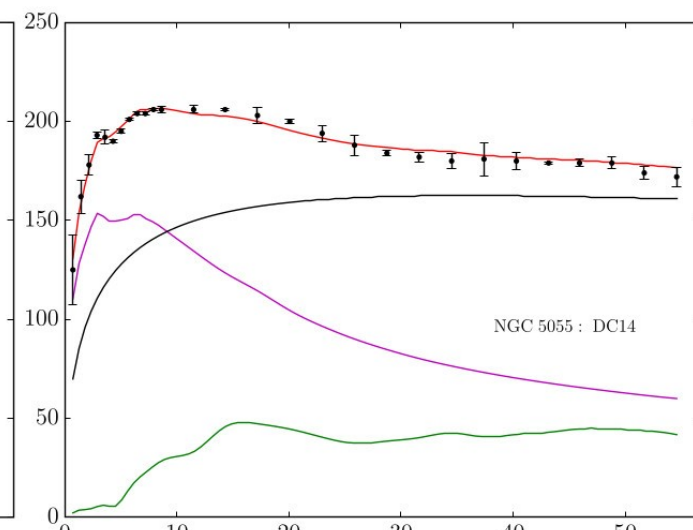
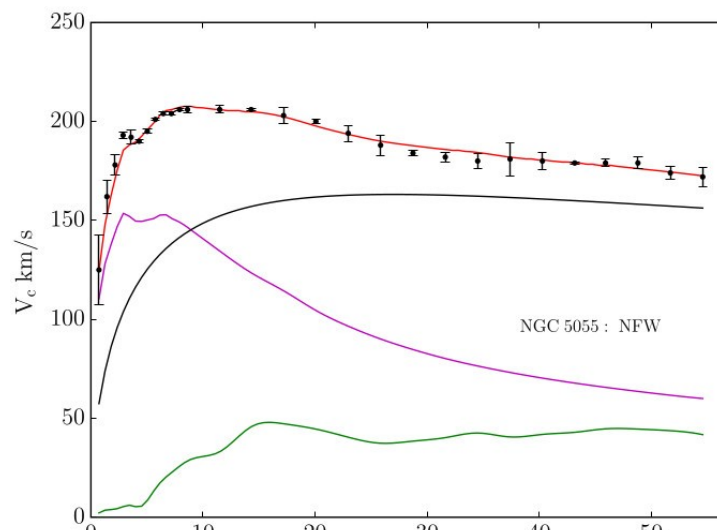
1. **Baryonic physics**: repeated, massive, and fast gas outflows due to stellar feedback produce non-adiabatic oscillations in the gravitational potential → transform DM cusp into a DM core

Baryon-modified DM halos

From Katz, Lelli et al. (2017)

Cuspy Halo (NFW)

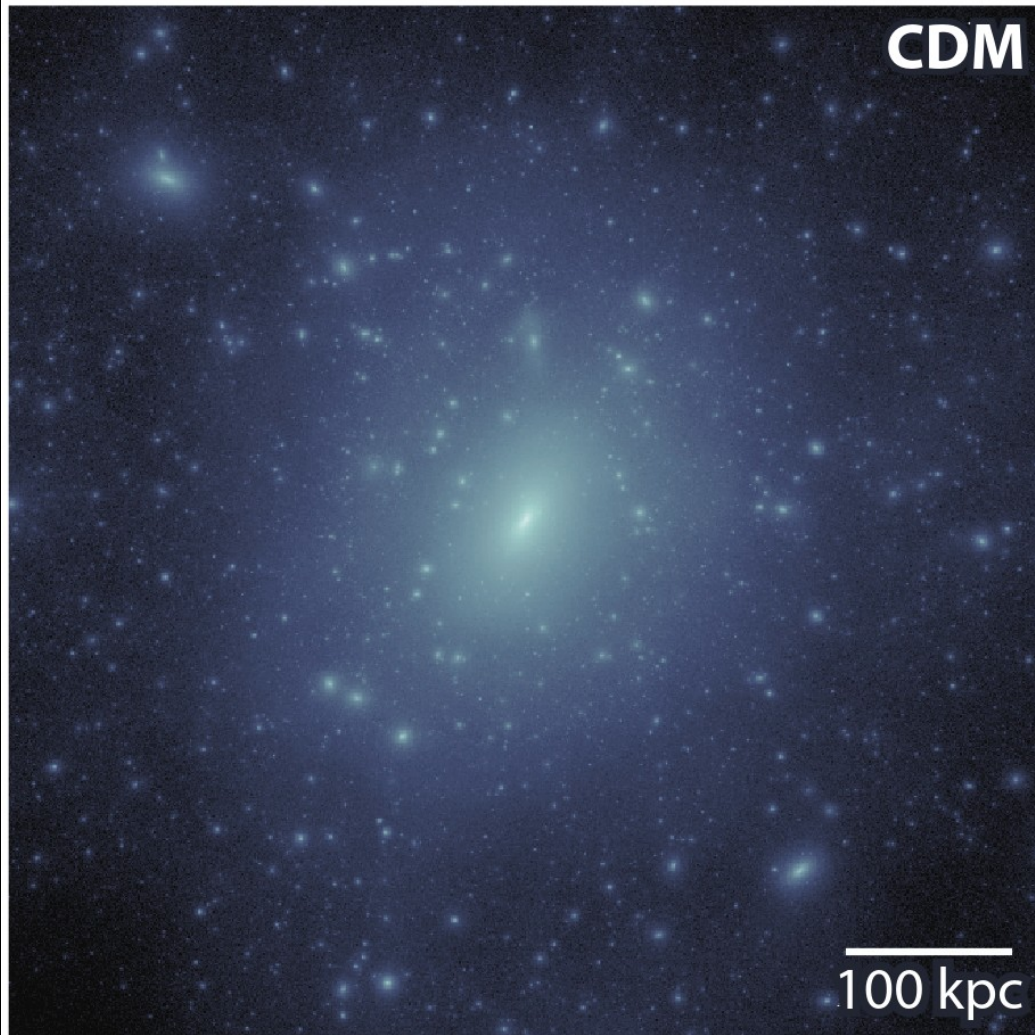
Baryonic-feedback-altered Halo



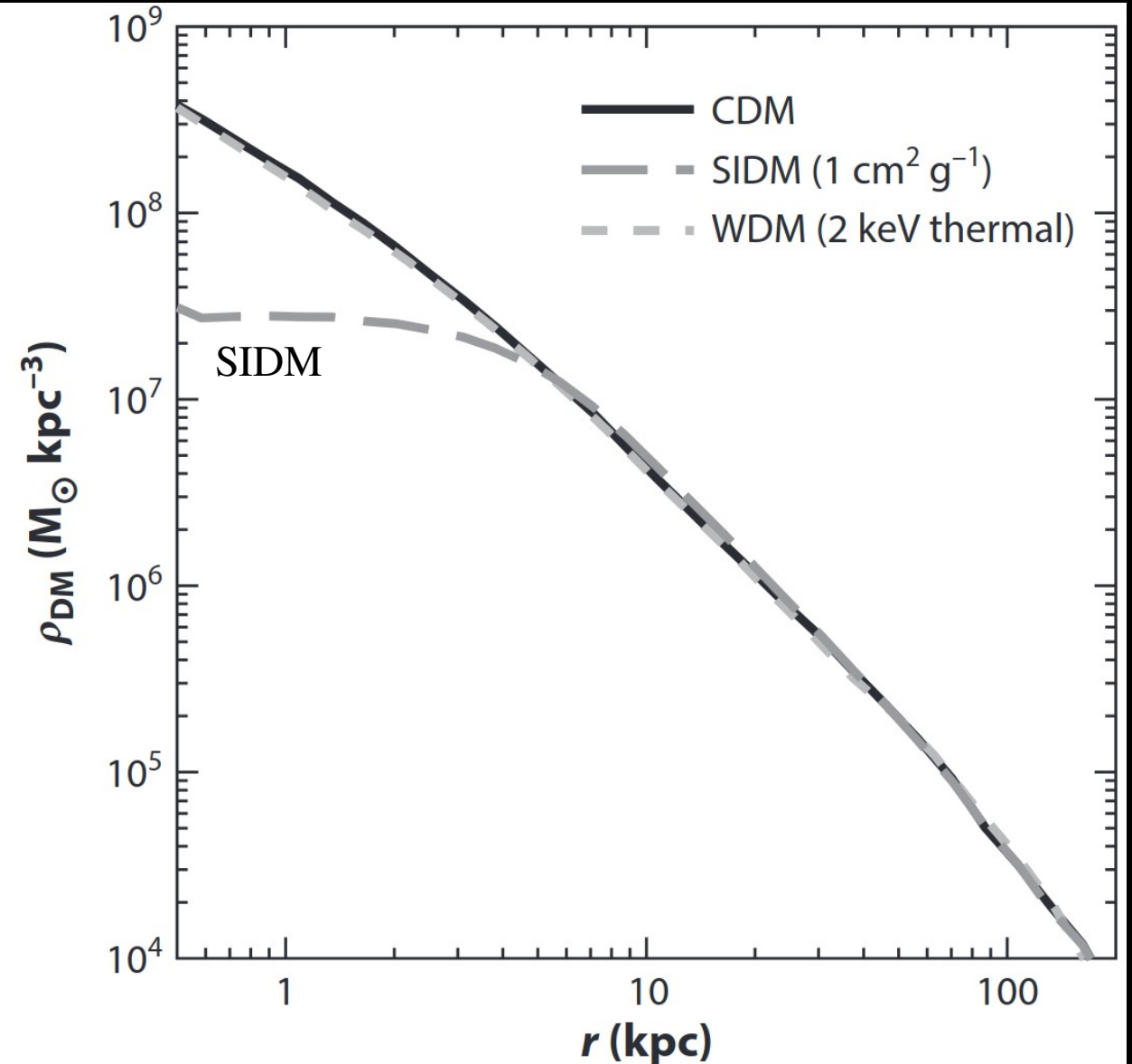
Possible Solutions to the Cusp-Core (Rotation Curve) Problem

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2. **Self-interacting DM (SIDM)**: cores are created because of the “pressure” from DM self-interactions (WDM don't work!)

Constant-density cores in SIDM



From Bullock & Boylan-Kolchin (2017)

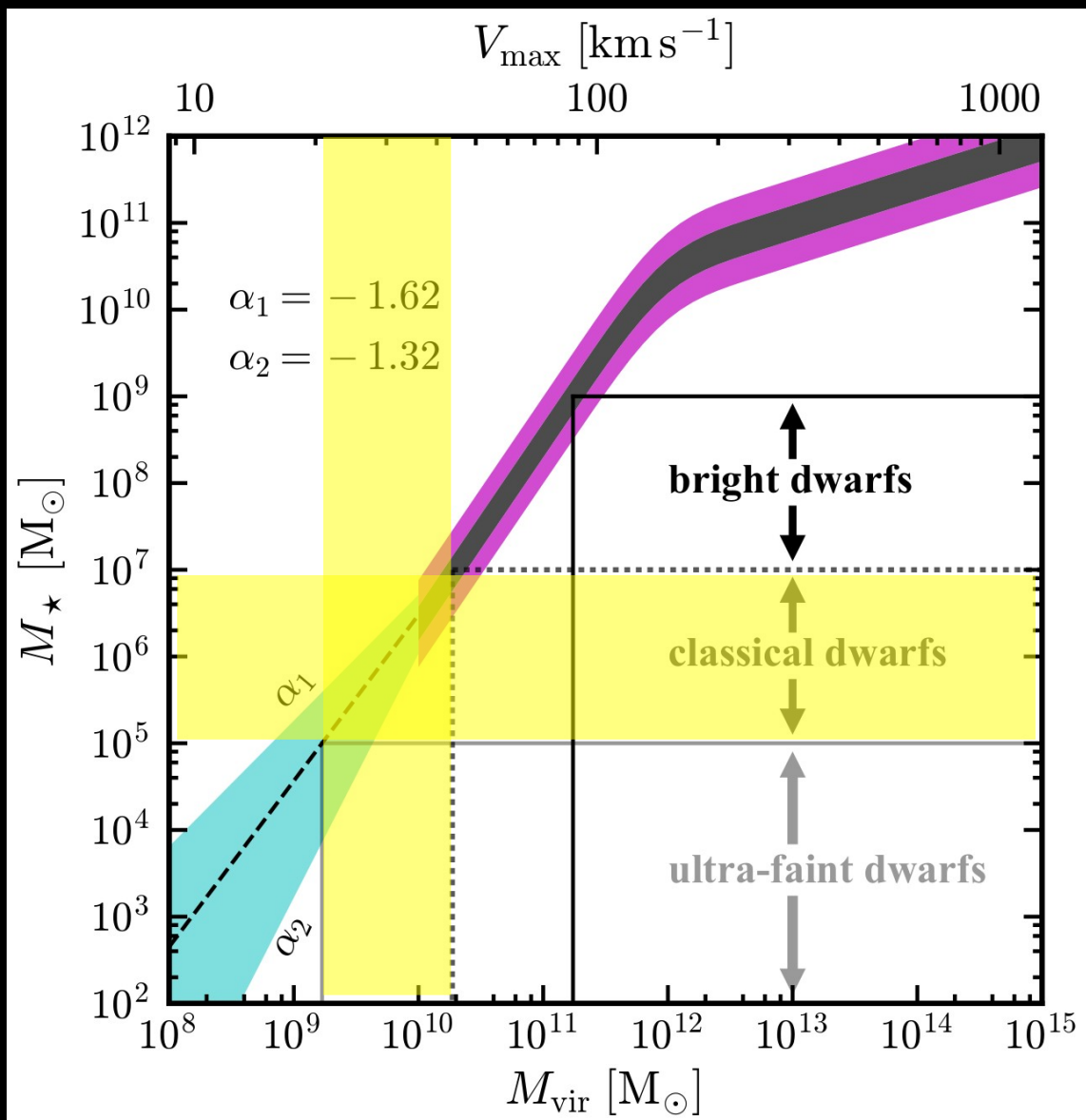


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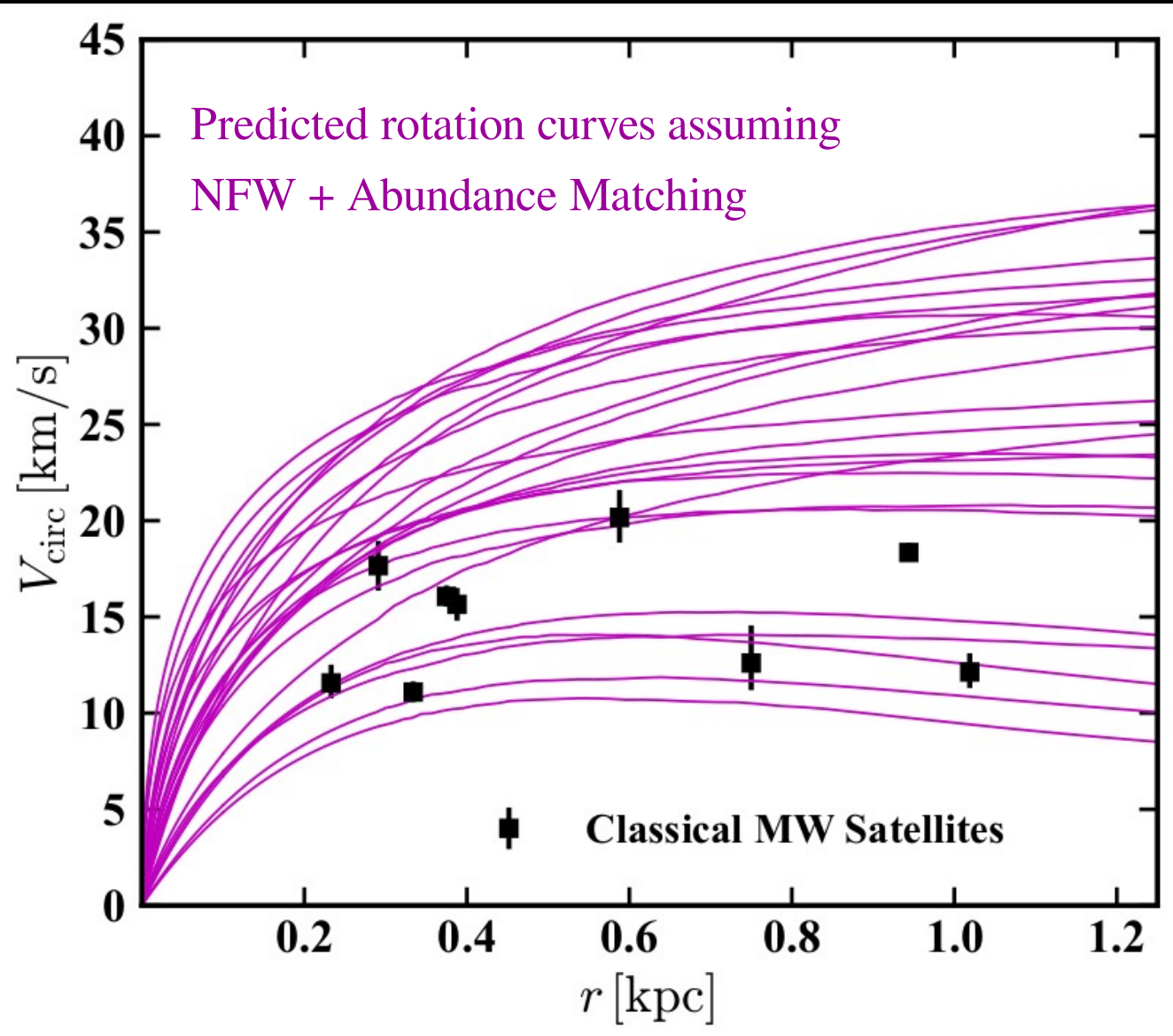
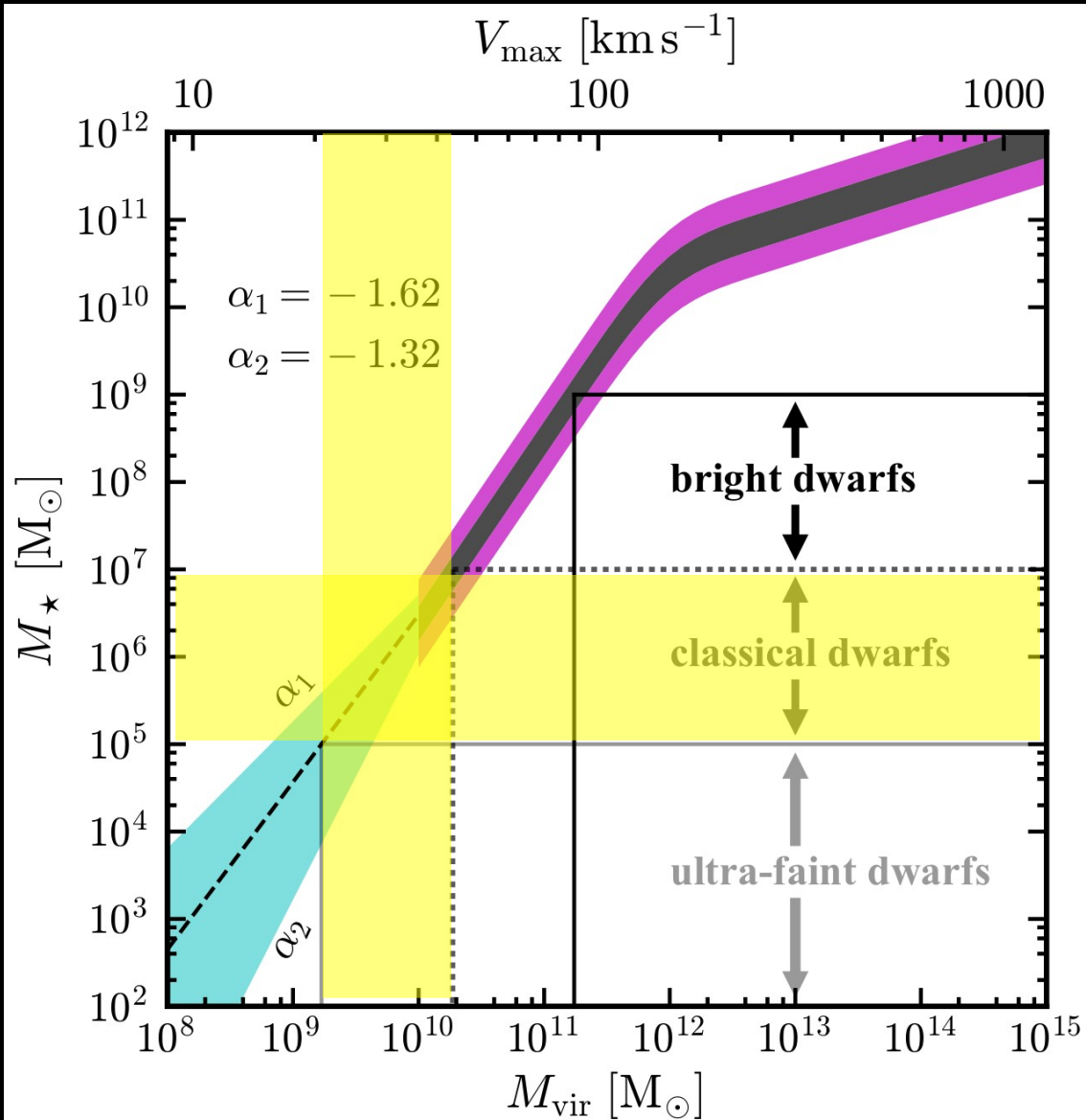
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3. **Modified gravity** (especially MOND): problem does NOT exist!
→ rotation curves are predicted from the baryonic distribution

3. Too-Big-To-Fail Problem (Problems 1+2 Reloaded)

Too-Big-To-Fail Problem



Too-Big-To-Fail Problem



Possible Solutions to the TBTf Problem (pushing down to very low-mass galaxies)

1. **Baryonic physics**: unclear whether it will work; stellar feedback should be inefficient in low-mass galaxies with $M_{\star} < 10^6 M_{\odot}$ because they produced very few supernovae!

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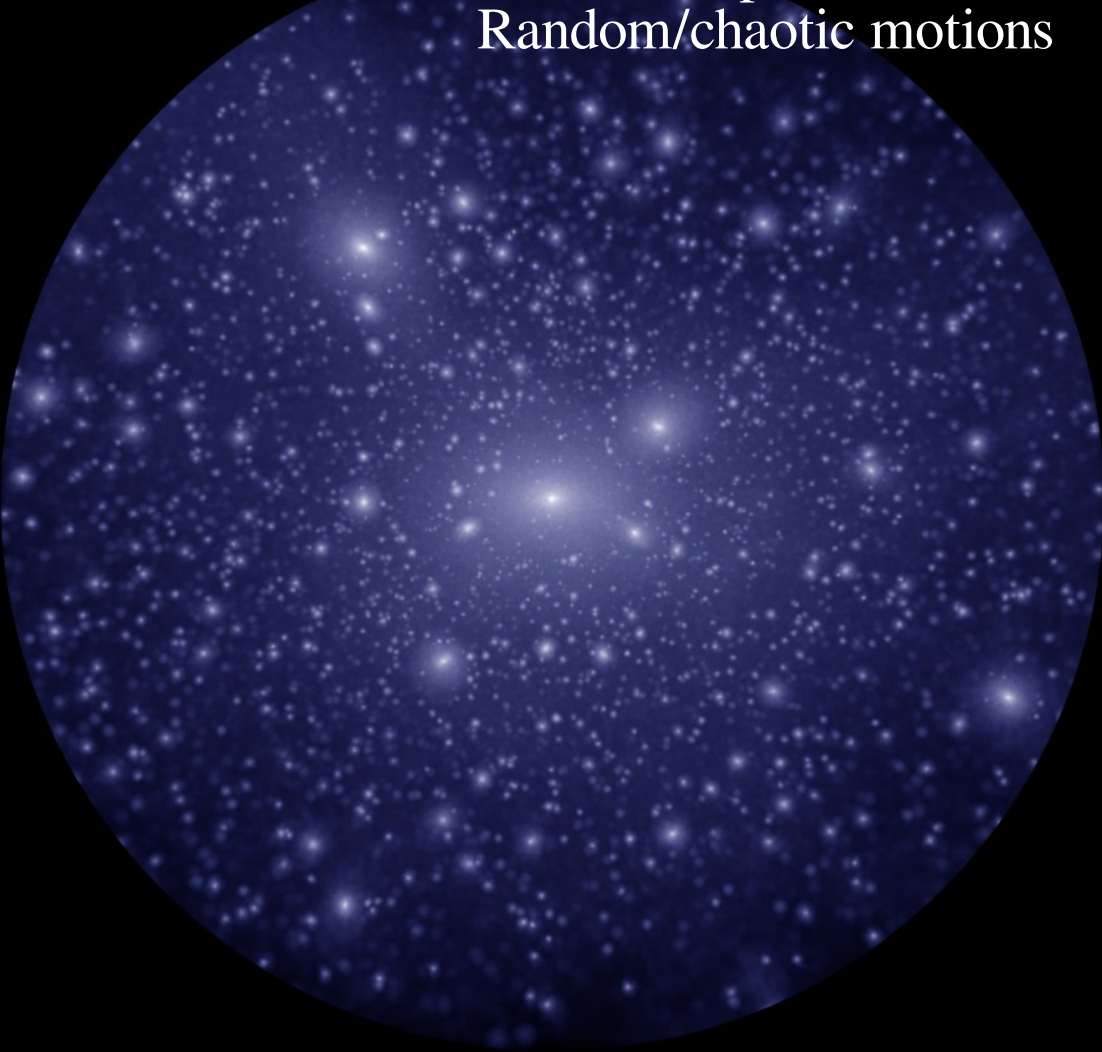
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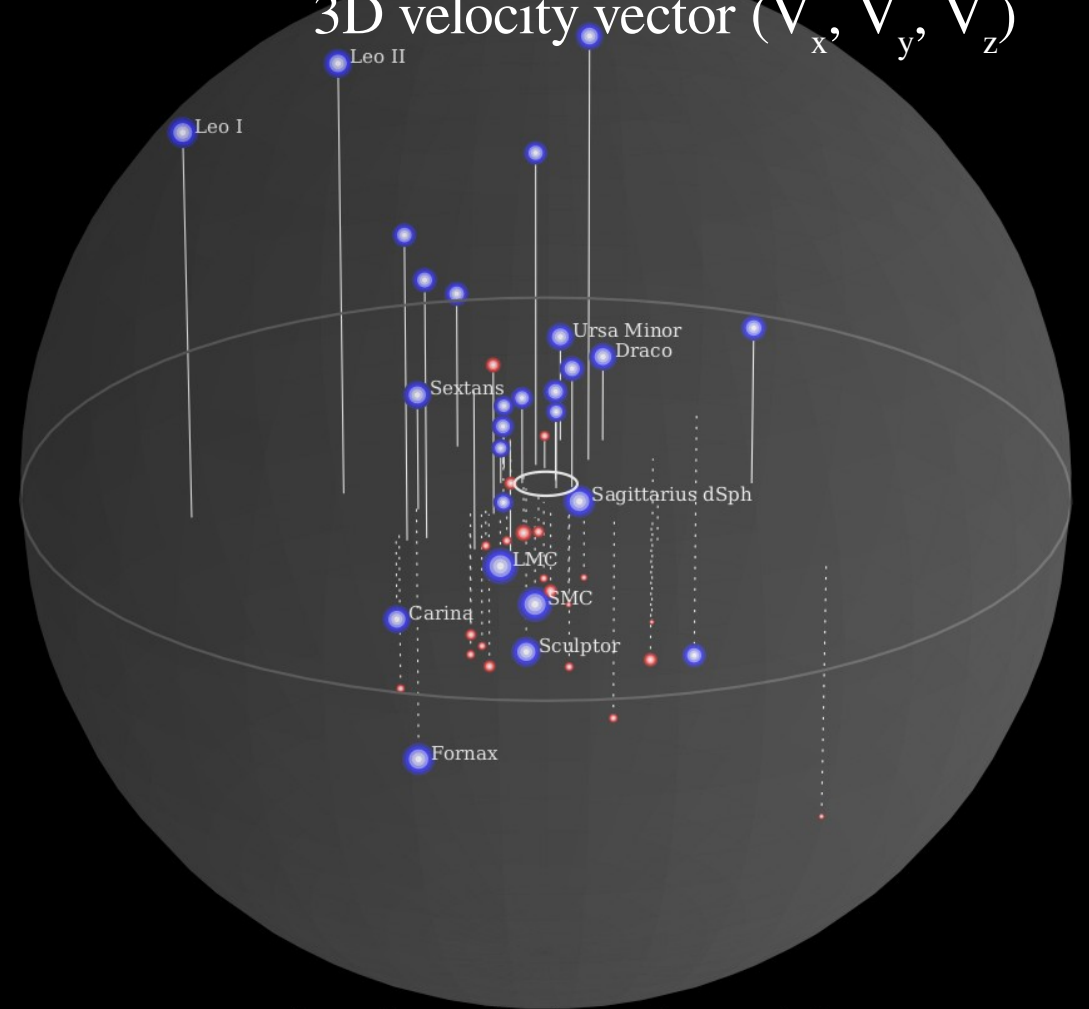
4. Planes of Satellites Problem (Satellites Phase-Space)

Distribution & Kinematics of Satellites

Λ CDM simulations: Near-isotropic distribution
Random/chaotic motions



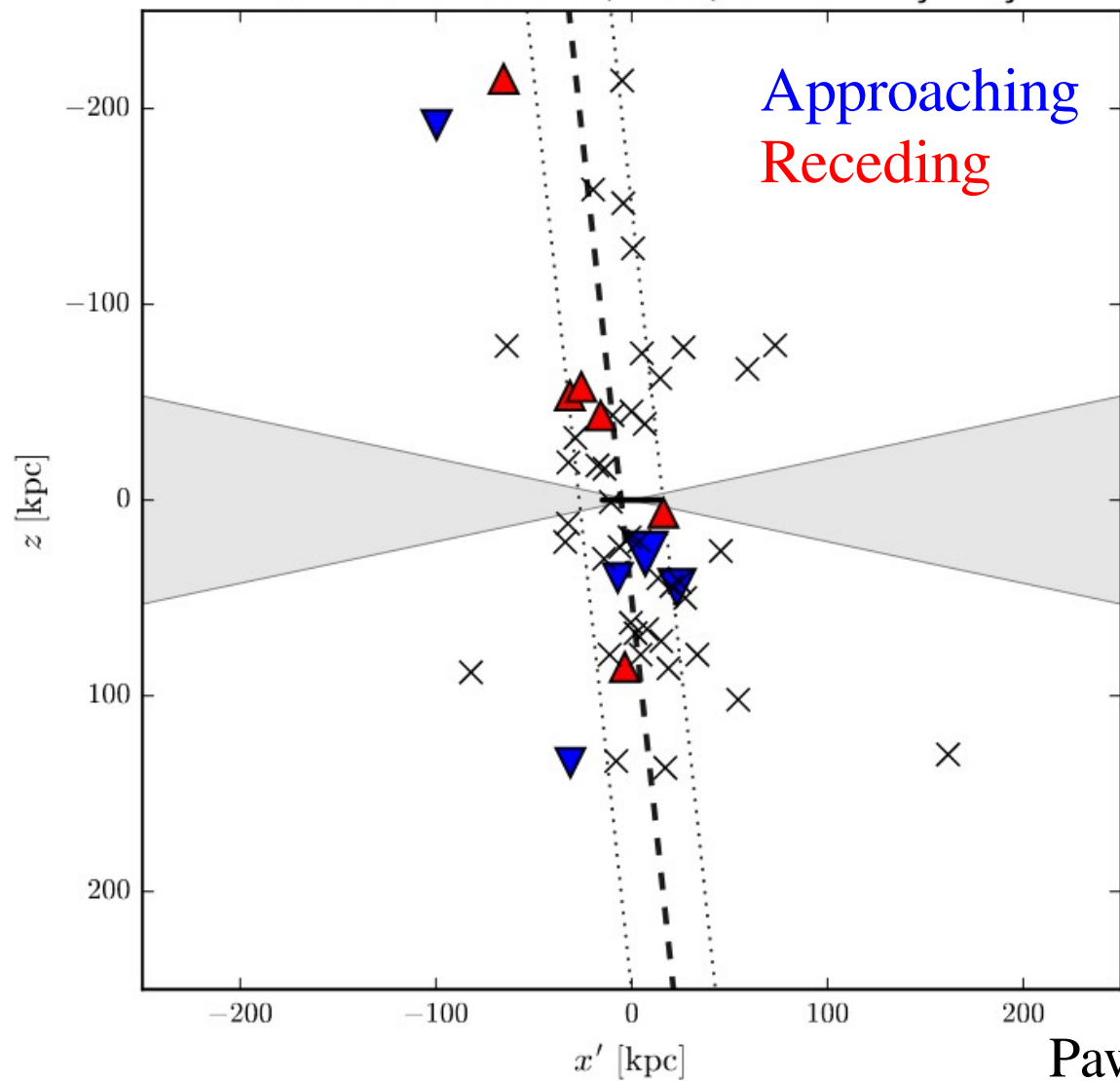
Observations: Need accurate distances +
3D velocity vector (V_x, V_y, V_z)



Pawlowski/Bullock/Boylan-Kolchin

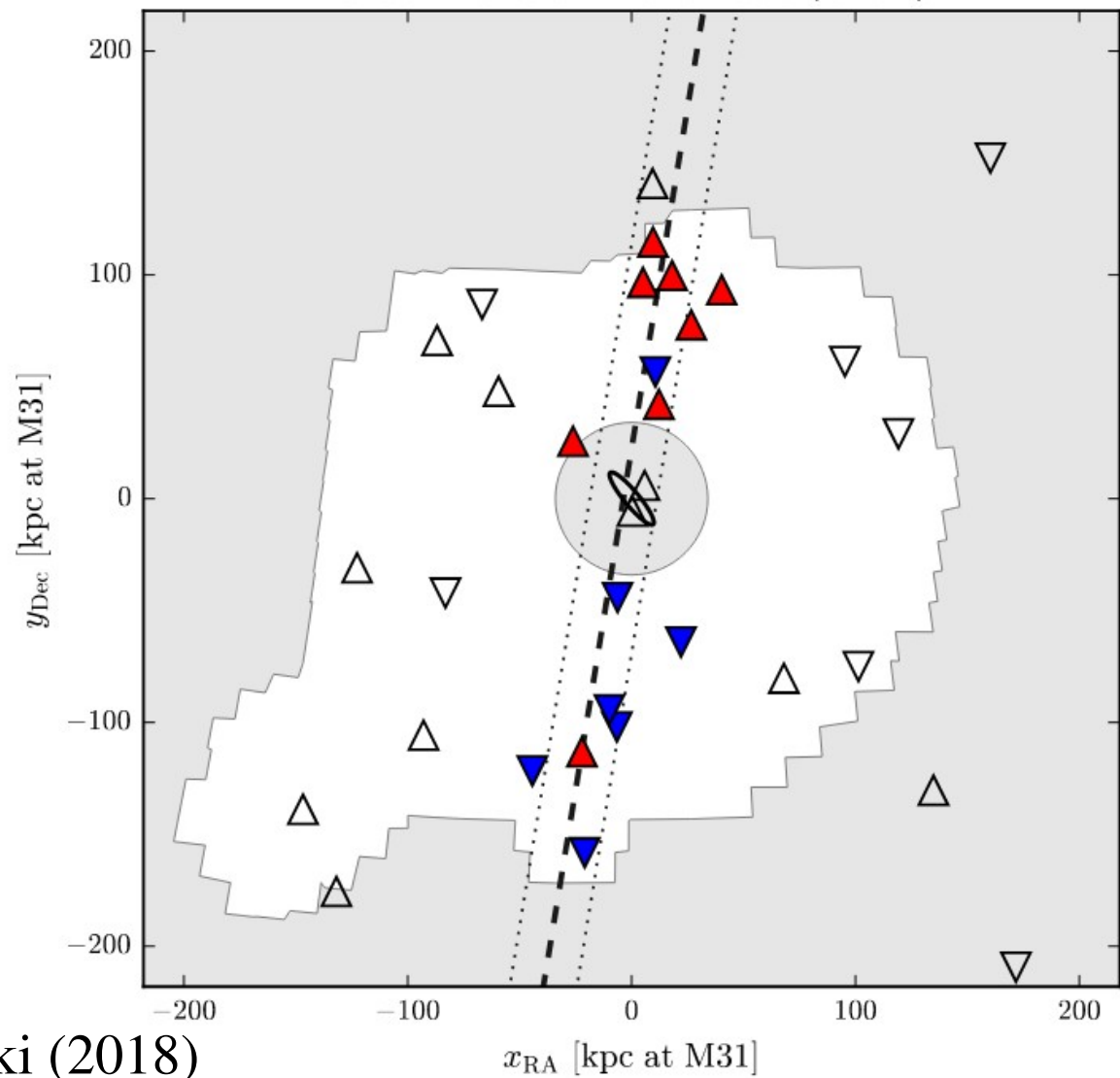
Planes of Satellites in the Local Group

Vast Polar Structure (VPOS) of the Milky Way

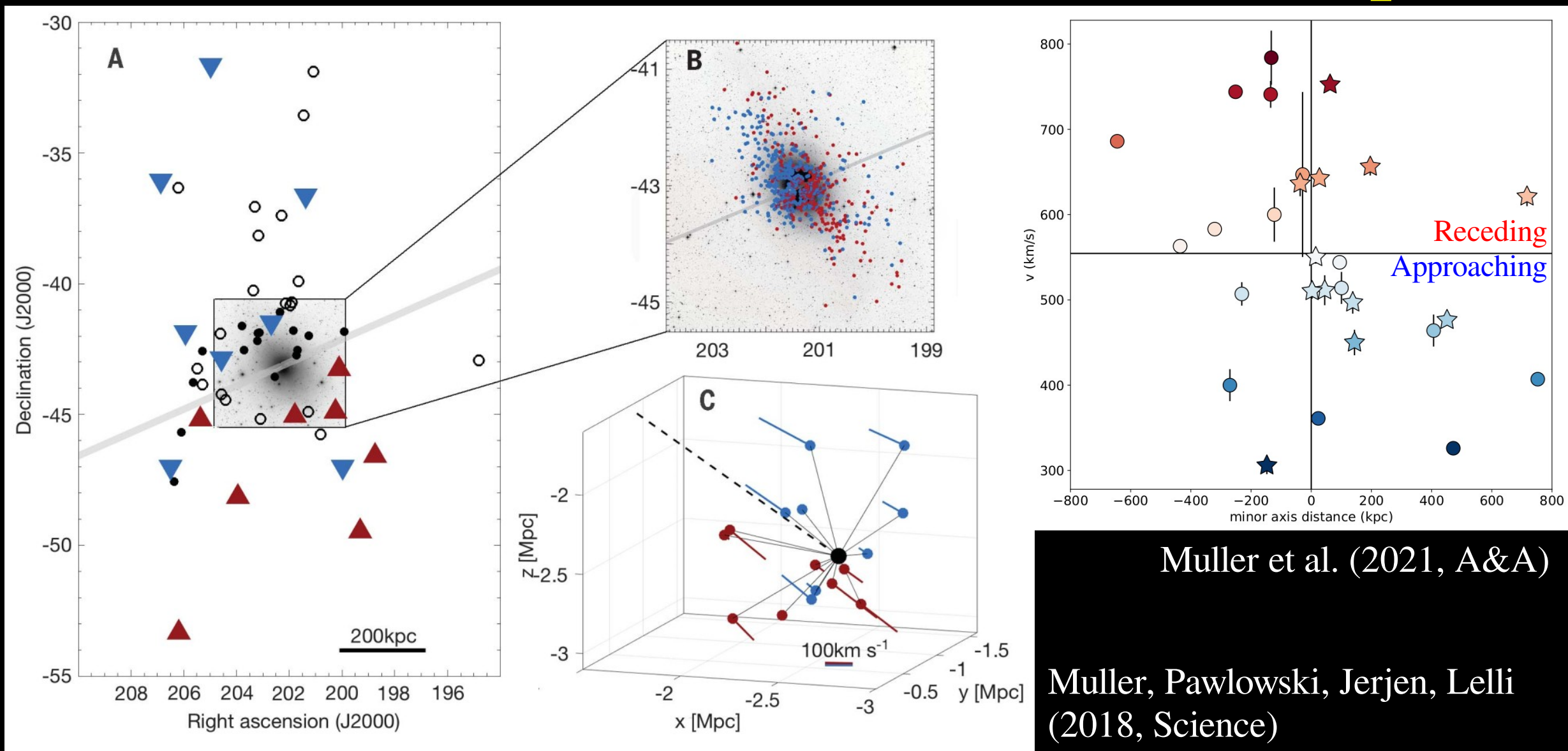


Pawlowski (2018)

Great Plane of Andromeda (GPoA)



Plane of Satellites in the Cen-A Group



Possible Solutions to the TBTF Problem (pushing down to very low-mass galaxies)

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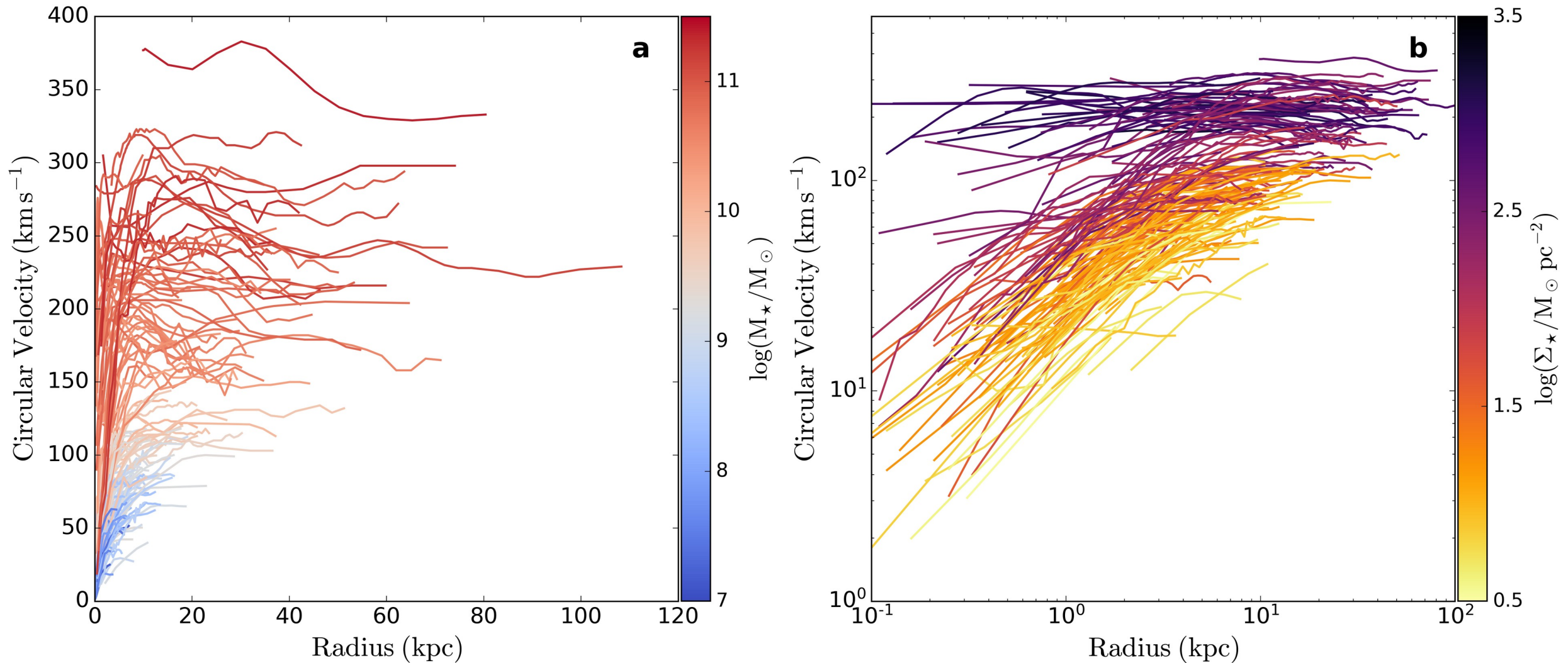
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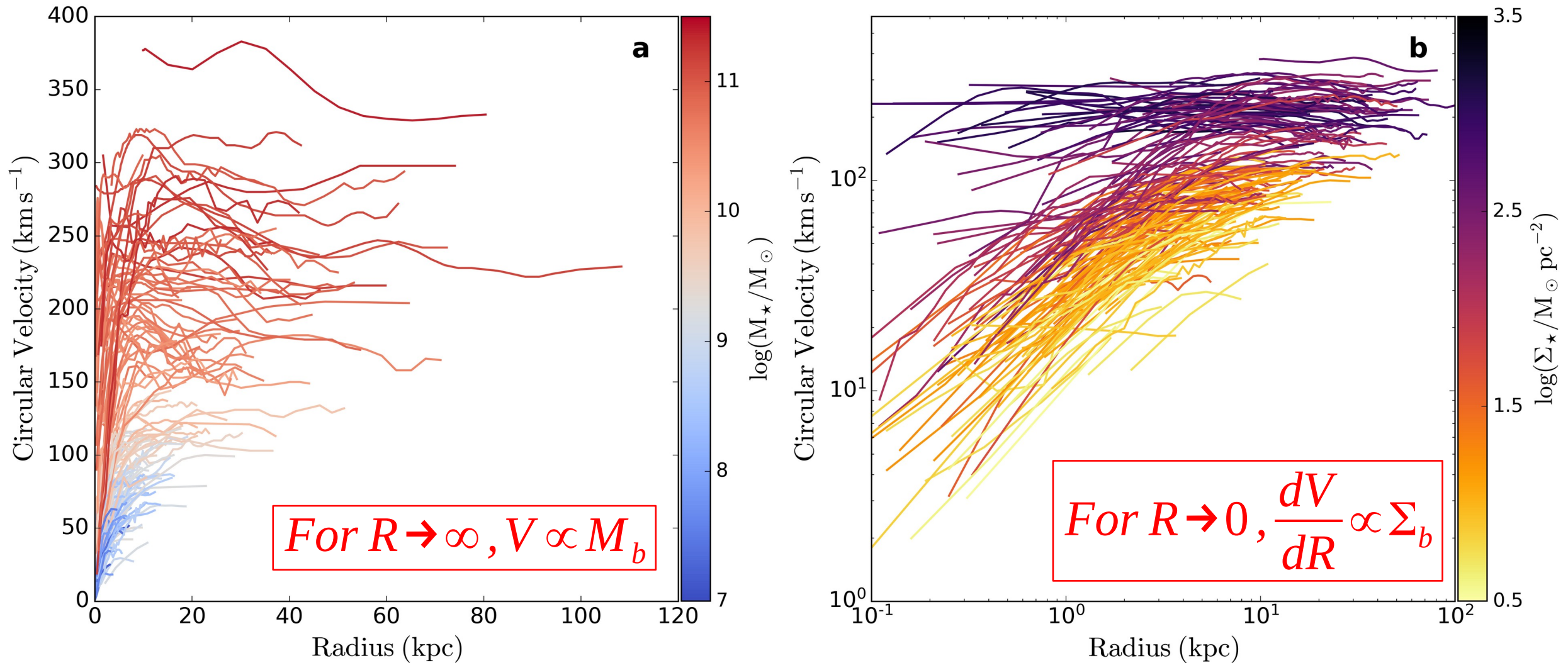
5. Dynamical Regularities (Baryon-DM Coupling)

Rotation Curves Shapes \leftrightarrow Baryonic Distribution



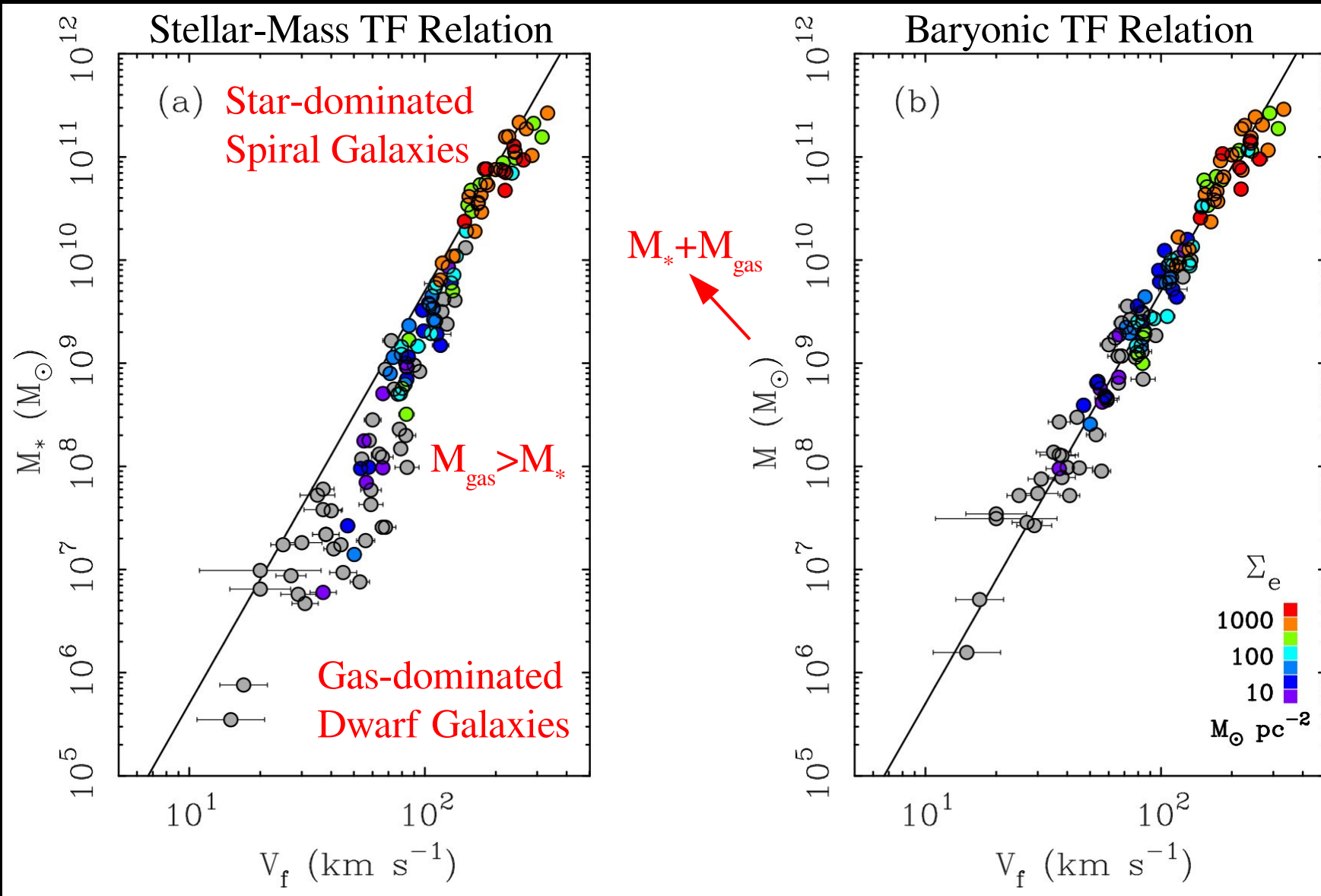
Lelli+(2022, Nature Astronomy)

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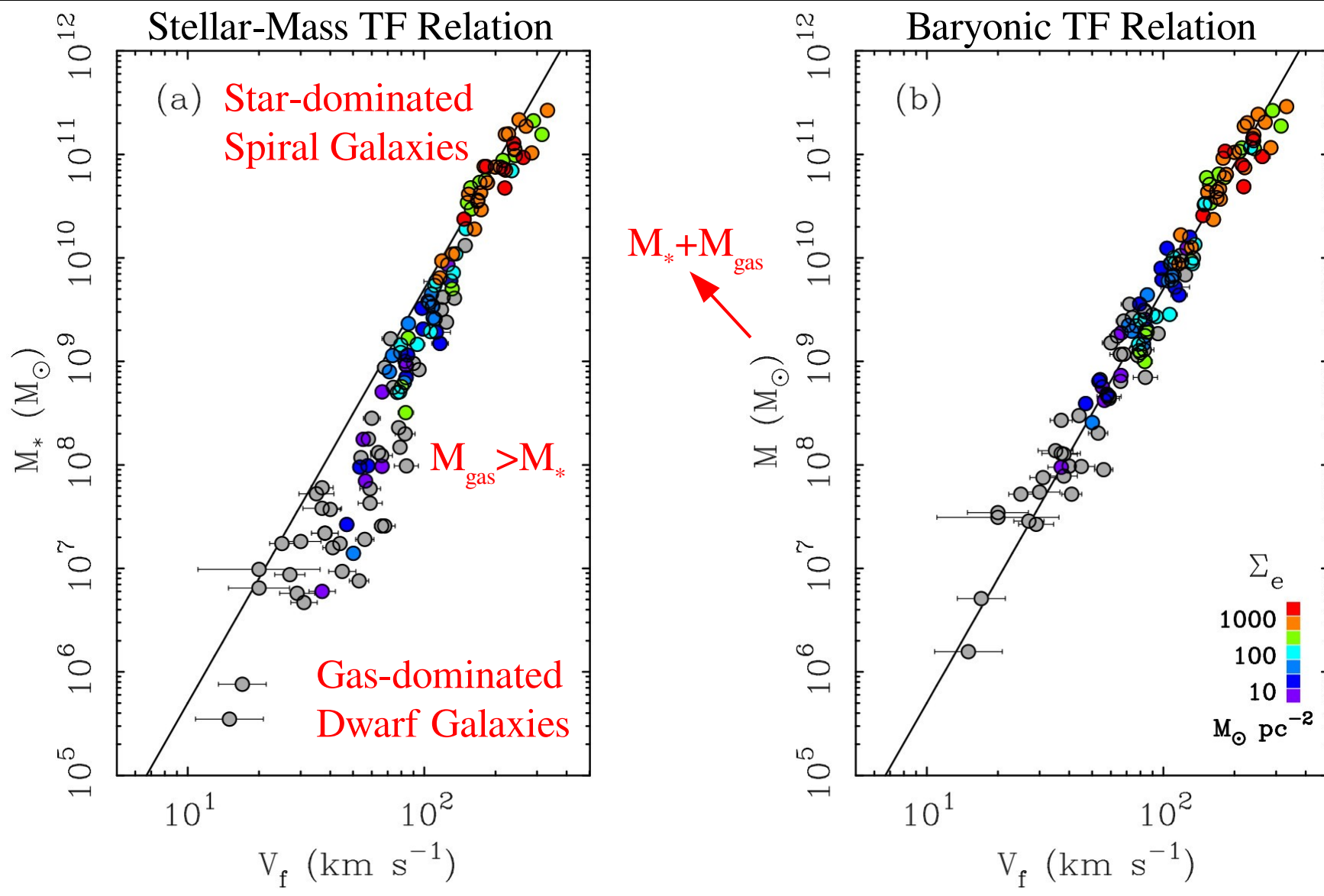


Lelli+(2022, Nature Astronomy)

Tully Fisher Relation: Mass versus Velocity



Tully Fisher Relation: Mass versus Velocity



Newton's Law gives:

$$V^4 = \frac{\pi^2 G^2}{f_b^2} \Sigma_b M_b$$

$$f_b = M_b / M_{\text{tot}}$$

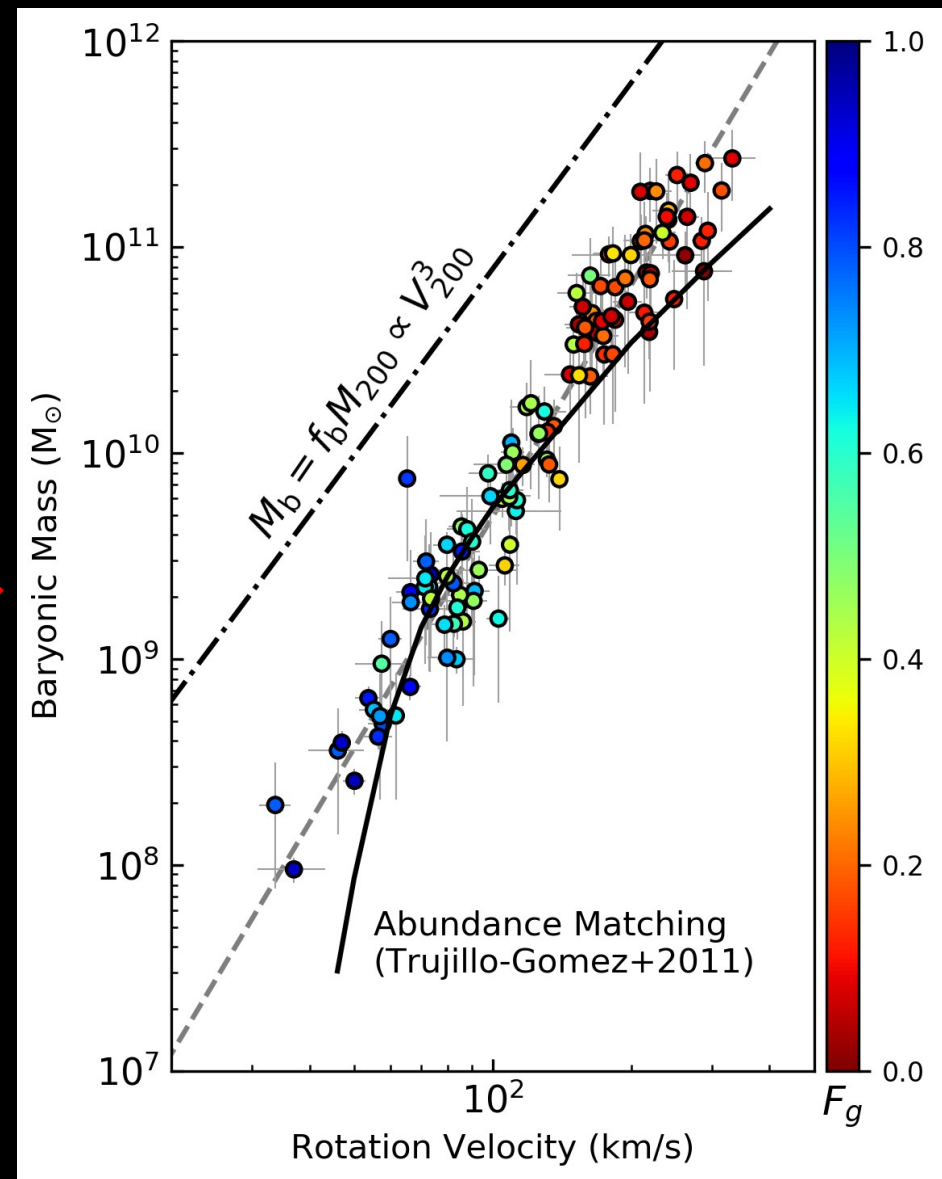
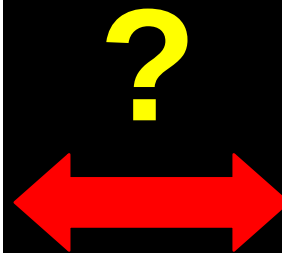
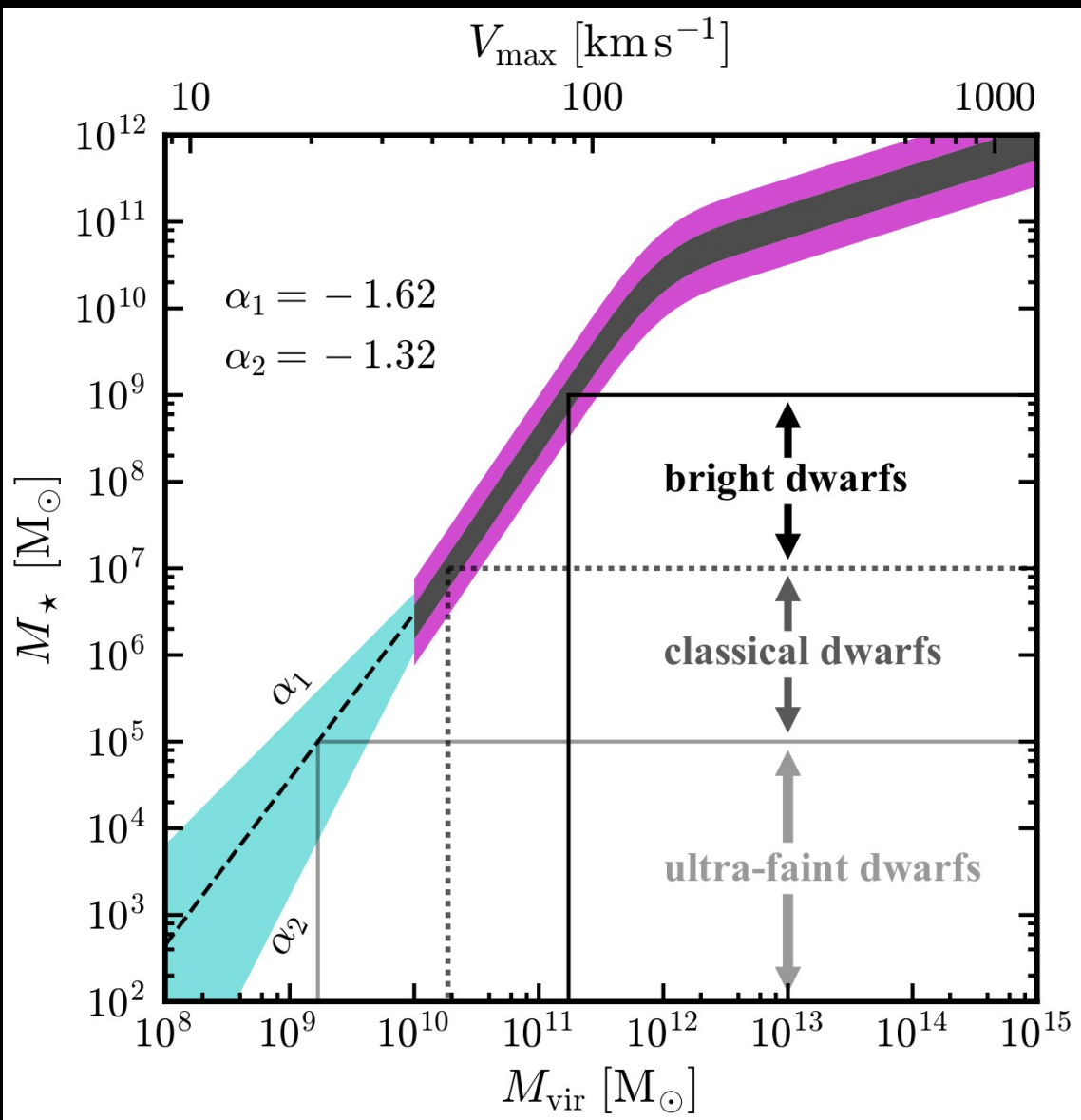
$$\Sigma_b = M_b / \pi R^2$$

But no dependence on Σ_b is observed.

$$\rightarrow \Sigma_b / f_b^2 \sim \text{const}$$

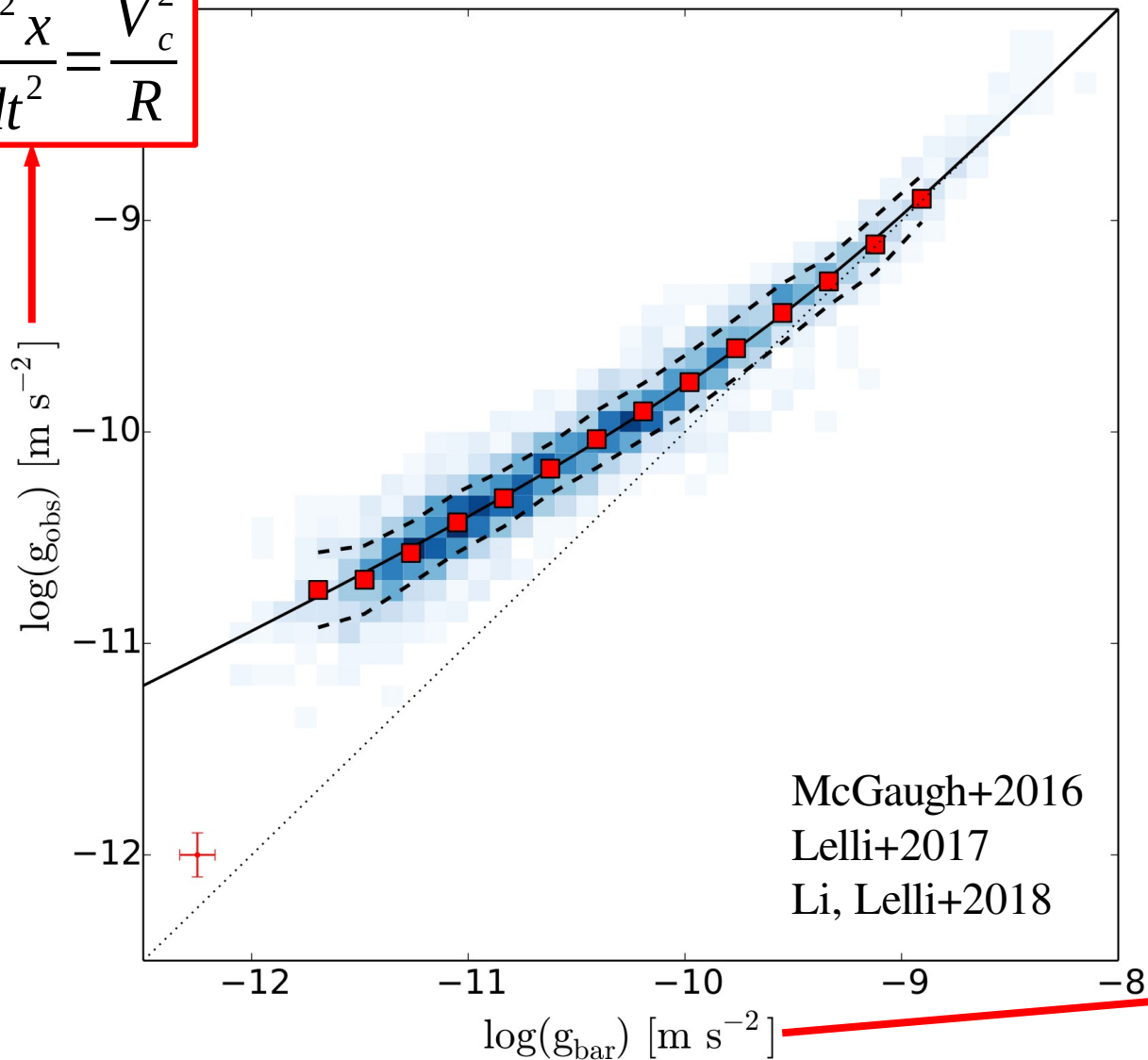
Fine-tuning problem

Abundance Matching \rightarrow Curved BTFR?



Radial Acceleration Relation (RAR)

$$\frac{d^2 x}{dt^2} = \frac{V_c^2}{R}$$



Observables:

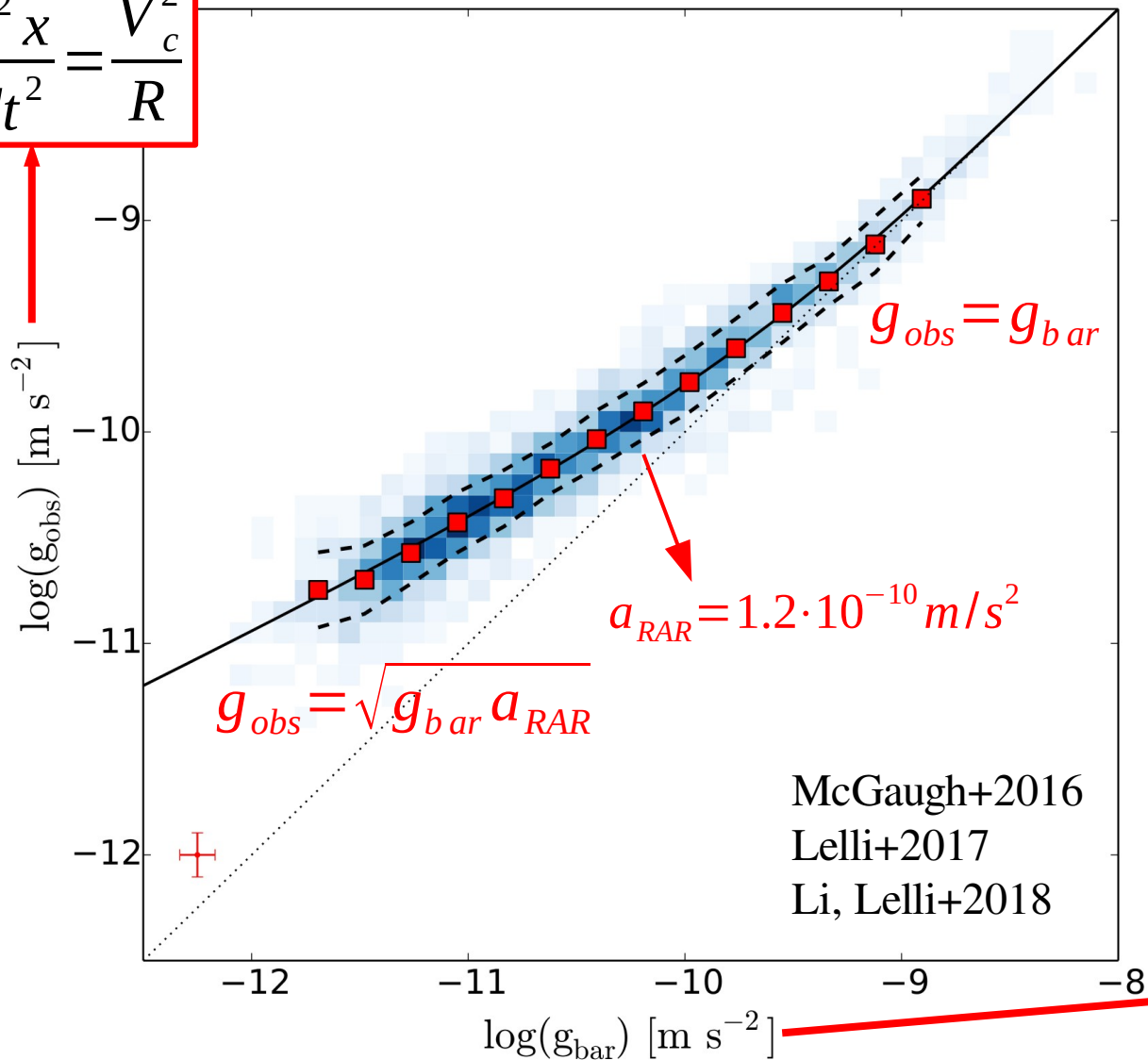
g_{obs} → centripetal acceleration from RCs

g_{bar} → gravitational field from baryons

$$g_b = -\nabla \Phi_b$$
$$\nabla^2 \Phi_b = 4\pi G \rho_b$$

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$$\frac{d^2 x}{dt^2} = \frac{V_c^2}{R}$$



Observables:

g_{obs} → centripetal acceleration from RCs

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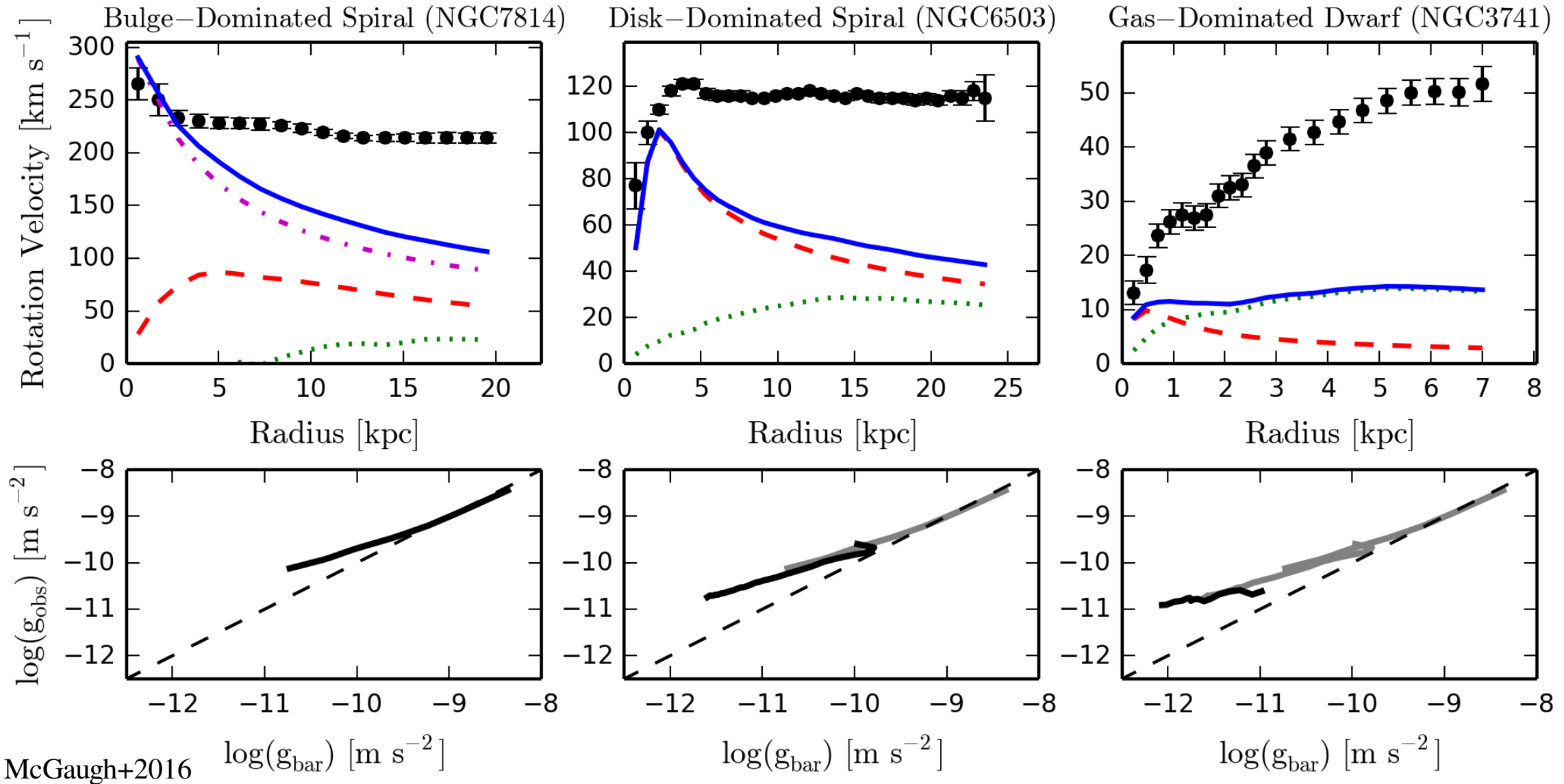
Key Properties:

- Acceleration scale $a_{\text{RAR}} \sim 10^{-10} \text{ m/s}^2$
- Small scatter (consistent with obs. errors)
- No residual dependencies (radius, etc.)
- Baryon distribution ↔ Rotation Curve

$$g_b = -\nabla \Phi_b$$

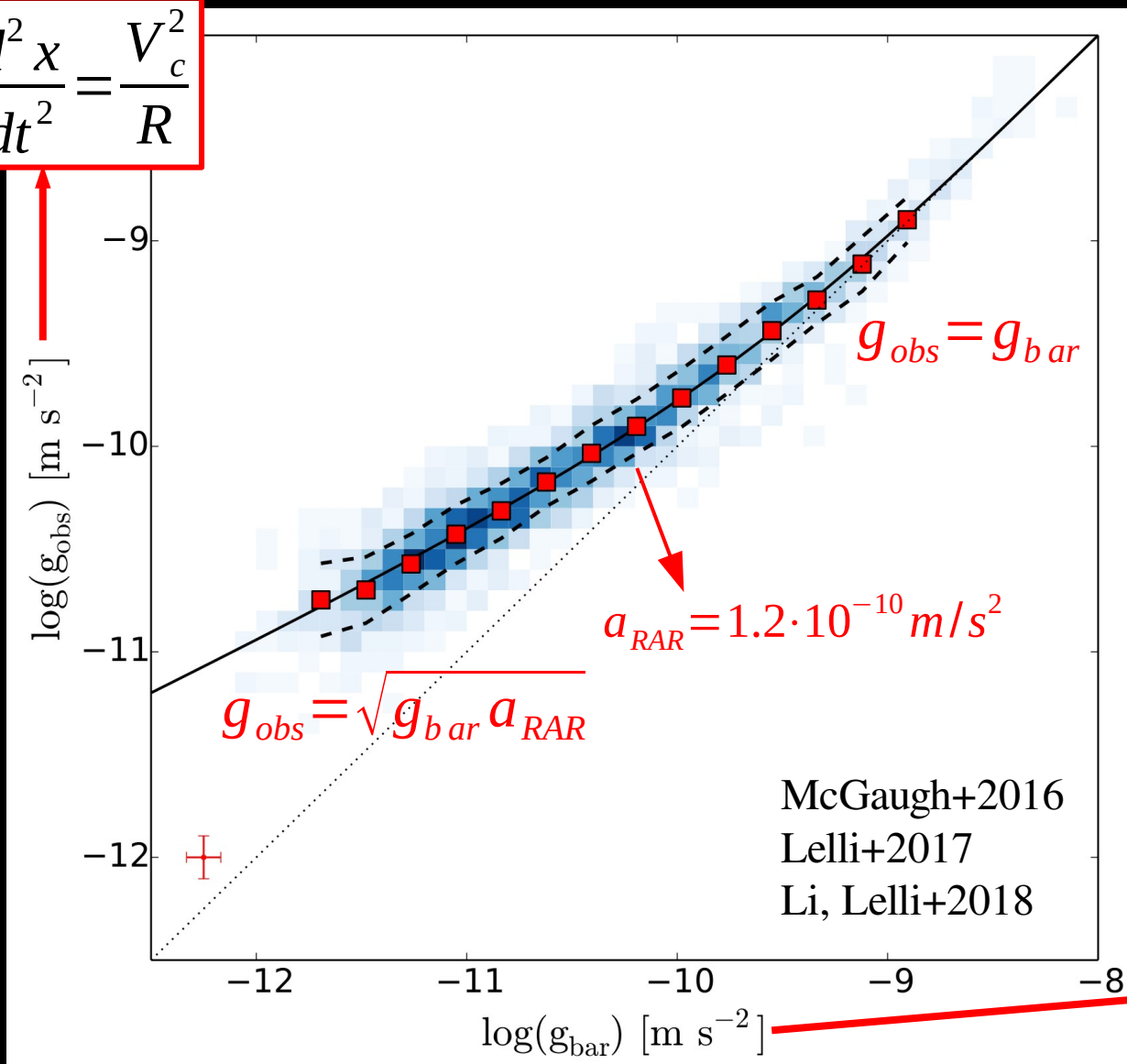
$$\nabla^2 \Phi_b = 4\pi G \rho_b$$

Very different galaxies on the same RAR



RAR sets the DM halo profiles

$$\frac{d^2 x}{dt^2} = \frac{V_c^2}{R}$$



$$g_{DM} = g_{obs} - g_{bar} = F(g_{bar})$$

$$M_{DM}(R) = \frac{R^2}{G} F(g_{bar})$$

No freedom to fit arbitrary DM halos!

“Cusp vs Core” is a symptom of a more serious general illness:

Baryon-DM coupling at any radii!

$$g_b = -\nabla \Phi_b$$

$$\nabla^2 \Phi_b = 4\pi G \rho_b$$

Small Scale Λ CDM Problems

1. Missing Satellites Problem

2. Cusp vs Core Problem

3. Too-Big-Too-Fail Problem

Baryonic physics in Λ CDM?

WDM+SIDM ?

Modified gravity (MOND)?

4. Planes of Satellites Problem \rightarrow ?

5. Dynamical Regularities \rightarrow Baryon-DM interaction? MOND?